GEIST

Torque Characteristics of Various Types of Electric Motors

Electrical Engineering

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TORQUE CHARACTERISTICS

OF VARIOUS TYPES OF ELECTRIC MOTORS

BY

HARRY FOREST GEIST

THESIS

FOR THE

DEGREE OF BACHELOR OF SCIENCE

IN

ELECTRICAL ENGINEERING

COLLEGE OF ENGINEERING

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May 30, 1912

THIS IS TO CERTIFY THAT THE THESIS PREPARED UNDER MY SUPERVISION BY

HARRY FOREST GEIST

ENTITLED TORQUE CHARACTERISTICS

OF VARIOUS TYPES OF ELECTRIC MOTORS

IS APPROVED BY ME AS FULFILLING THIS PART OF THE REQUIREMENTS FOR THE

DEGREE OF BACHELOR OF SCIENCE IN ELECTRICAL ENGINEERING

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21931

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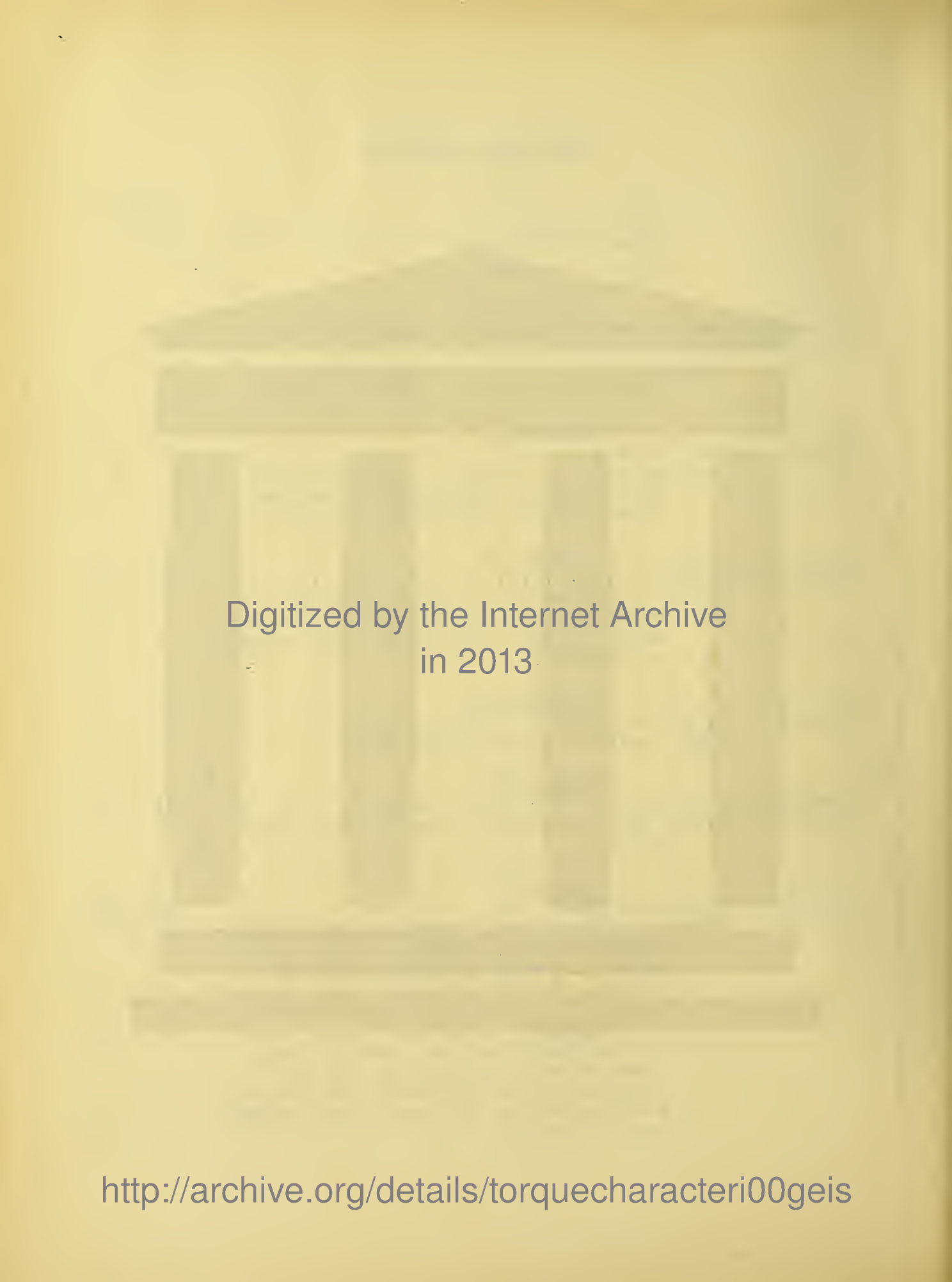
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Key to Symbols

I = current in general

I_a = armature current

I_f = field current

I_s = secondary current

I_p = primary current

E_x = impressed electromotive force

E_c or e = counter electromotive force

Z = number of conductors in series

M = stationary magnetic field

m = magnetic field about conductors carrying (current

Φ = flux

β = induction

F = force

T = torque

T_s = starting torque

p = pairs of poles

N = speed

P = power

$Z = \frac{2Zp}{p' \times 10^8}$, where p' equals conductors in (parallel

R_a = armature resistance

r_s = secondary resistance

r_p = primary resistance

X_s = secondary reactance

X_p = primary reactance

Key to Symbols (continued)

S	= slip
g	= admittance to coreloss current
b	= admittance to magnetizing current
f	= frequency
Z_s	= secondary impedance
Z_p	= primary impedance

PREFACE

The purpose of this thesis is to study the torque of the direct current shunt motor, of the direct current series motor, and of the alternating current polyphase induction motor.

The scope of the thesis will be a complete study of the fundamental principles of the machines under consideration, derivation of the fundamental formulae, and to show by calculated results and graphs the relation between load, current, torque, speed, losses, and efficiency, with special attention to the relations of torque and current.

Besides the true efficiency, which equals the input minus losses divided by input, the torque efficiency will be considered in connection with torque. True efficiency is calculated from the consideration of the losses, while torque efficiency, which by definition equals starting torque times normal speed divided by power input involved, will be calculated from torque, and comparison made by graphs between true efficiency and torque efficiency as well as between running torque and starting torque.

The study of torque and its relation to speed and power developed is a very helpful one to the electrical engineer, because of the wide field in the industrial world for electric motors. The intelligent application of motors must be based upon a thorough knowledge of what the machine will do under all conditions, and this behavior can best be determined

by a study of characteristics. A thorough knowledge of torque and the factors that produce it and their relation to highest perfection in motors is also of importance to the designing engineer. It is with these things in mind that I respectfully submit this thesis.

Harry Forest Geist

INTRODUCTION

Electro-magnetic Transformation of Electrical to Mechanical Energy

If a current "I" is forced through a conductor by an electromotive force "E_x", a magnetic field "m" will be set up around the conductor. This field will encircle the conductor in a direction depending upon the direction of the current through the conductor. The rotation is clockwise with a receding current, and vice versa as shown by diagram #1.

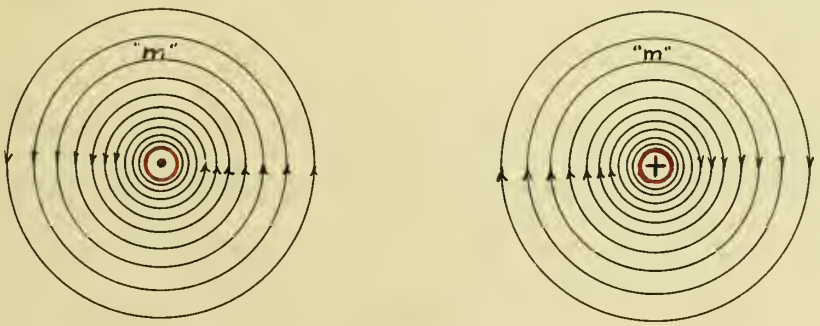


Diagram #1

If the conductor is placed in another magnetic field "M", and no current forced through it, the field "M" will not be disturbed, except for the slight difference in permeability of the conductor material and that through which "M" flows. But by forcing a current through the conductor, the added effects of "M" and "m" cause a distorted field, as is shown by diagram #2.

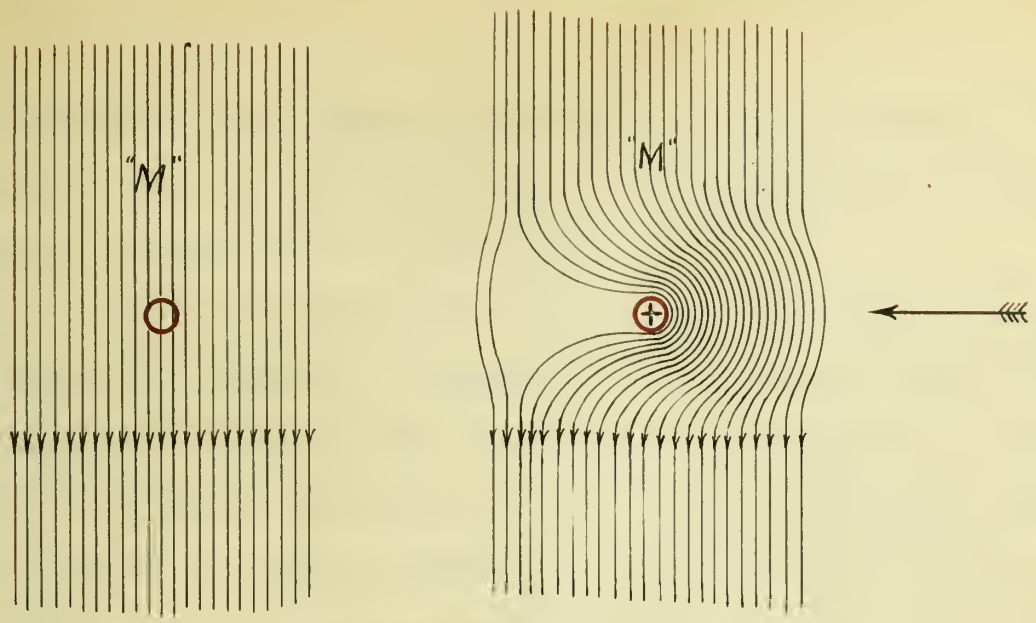


Diagram #2

The cause of distortion is the fact that the magnetic field "m" combines with the flux "M" on one side of the conductor, and opposes it on the other side. Due to the fact that fluxes in the same direction add together and form a flux equal to their sum, and opposing fluxes neutralize each other and leave a flux equal to their difference, it is evident that the conductor will now have a comparatively strong field on one side and a weak field on the opposite side. This causes a tendency to move the conductor in a direction to the side with the weak field which is at 90° to the field "M", as shown by diagram #2.

The force tending to move the conductor depends upon the strengths of "M" and "m", but since "m" varies directly as the strength of "I" flowing through the conductor, it is preferable to consider the force upon the conductor as dependent upon "M" and "I". If there are "Z" conductors, all with the same current "I" flowing in the same direction, it is evident that if "M" is constant, that the force upon the conductors will be

"Z" times the force upon one conductor. It is essential to note here that for the same force it is possible to vary the relation between "M", "I", and "Z".

The same force that causes the conductor to move in the field "M" causes the conductor to cut across lines of flux in its path. Lines of flux are elastic, so that when a conductor cuts across them they tend to stretch around the conductor until it passes through. The flux is given a sort of rotating motion about the conductor, which induces an electromotive force E_c in the conductor in opposition to the impressed electromotive E_x. In other words, like every other phenomenon in nature, so also in electricity we find that results tend to destroy their cause. This induced electromotive force "E_c" is called the counter electromotive force. It is dependent for its value upon the number of lines of flux "Φ" that the conductor cuts, the number of conductors "Z" cutting, and the speed "N" at which the lines of flux are cut.

The force then at any speed upon the conductors, is proportional to the flux "Φ" of field "M", the number of conductors "Z", and the current "I" forced through the conductors by the difference in voltages $E_x - E_c$. The force at starting is proportional to the flux "Φ", the number of conductors "Z", and the current "I" forced through the conductors by "E_x", because at starting "N" and consequently "E_c" equals zero.

CHAPTER I

Direct Current Motors

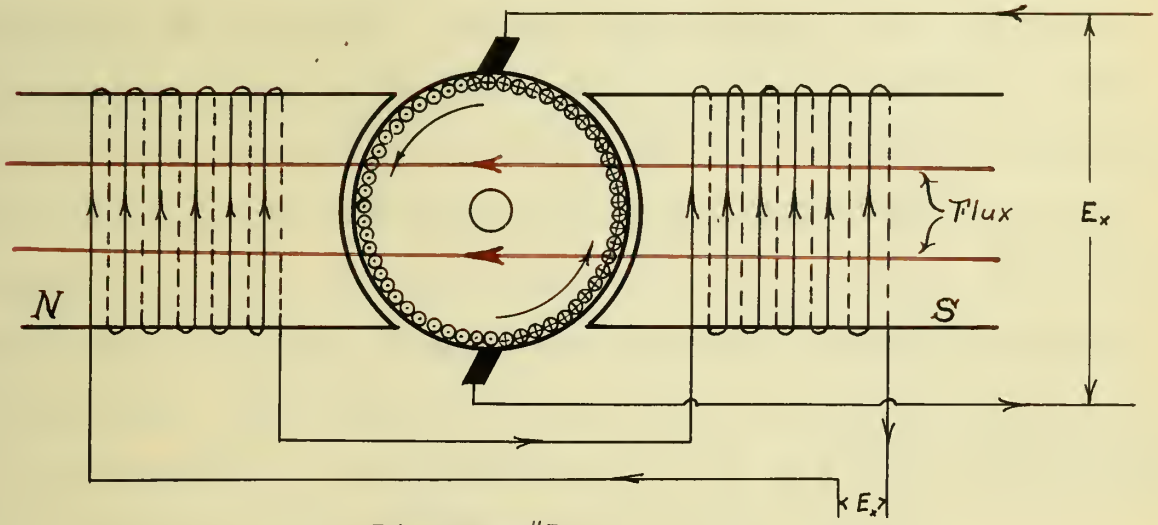


Diagram #3

The above diagram is a simple representation of the practical application of the transformation of electrical to mechanical energy as used in direct current motors. The flux "Φ" of field "M" passes from the iron core of the north pole, through an air gap, across the armature core and conductors, through another air gap, into the core of the south pole, and back to the north pole by an iron yoke not shown in diagram. The flux is set up by a conductor wound about the field core projections. Its strength depends upon the number of turns, the current flowing through the turns, and the permeability of the magnetic circuit. The armature is an iron cylinder mounted upon an axis, about which it is free to rotate. The conductors are wound upon the cylinders' outer surface, usually in slots,

parallel to the axis , in a continuous circuit so that the same current flows through all conductors. In order that we may have rotation it is necessary that the current in the conductors on the armature acts in the same relative direction with the flux "Φ" throughout the whole periphery of the armature. Since flux first enters and then leaves the armature in its path, it is evident that on one side of the armature, current must flow in one direction, and on the other half in the opposite direction. This function is performed by a commutator and brushes at the points where electrical energy enters the conductors. Now applying the principles explained in the preceding chapter to the diagram #3, it is seen that the armature will rotate in the direction shown by the arrow.

Since a motor is made for the purpose of delivering mechanical power, it is very important to study the mechanical relation between the electrical input and the mechanical output, so that if the electrical conditions are known, the mechanical behavior can be predetermined. The mathematical relation is based upon the law that the force exerted upon the armature equals the flux times the current times number of conductors; or what amounts to the same thing, the induction times the current times the length of conductor times the number of conductors, where induction "β" equals total flux divided by the area through which it flows.

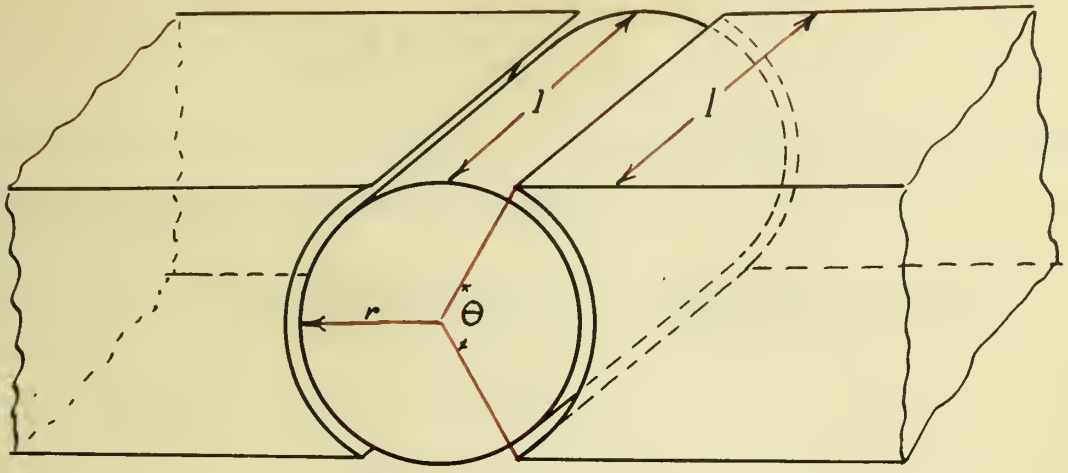


Diagram #4

If "θ" is the polar embrace, "r" the radius to center of conductors, and "l" the active length of conductor in centimeters, then the area under a pole face on the armature equals,

$$area = \frac{\theta \times 2\pi r \times l}{2\pi} = \theta r l$$

$$"\beta" = \frac{\Phi}{area} = \frac{\Phi}{\theta r l} \quad \text{lines of flux}$$

The force of each conductor under the pole face is then

$$"F" = \beta l I \times 10^{-7} \quad \text{dynes} \quad \text{where I is in amperes}$$

or $$"F" = \frac{\Phi I}{r \theta} \times 10^{-7} \quad \text{dynes}$$

Since the polar embrace is "$\frac{\theta}{2\pi}$", the number of conductors in series under a pole at any instant equals "$\frac{\theta}{2\pi} Z$". Also since each conductor carries the same current I, the total force exerted upon the armature by one pole equals

$$"F" = \frac{Z \theta}{2\pi} \times \frac{\Phi I}{r \theta} \times 10^{-7} \quad \text{dynes}$$

$$\text{or } \text{"}F\text{"} = \frac{Z \Phi I}{2\pi r} \times 10^{-1} \text{ dynes}$$

This force acts at a radius of "r" centimeters, so that the torque in dyne-centimeters equals

$$\text{"}T\text{"} = \frac{Z \Phi I r}{2\pi r} \times 10^{-1} = \frac{Z \Phi I}{2\pi} \times 10^{-1} \text{ dyne cm.}$$

For a two pole machine therefore

$$\text{"}T\text{"} = \frac{Z \Phi I}{\pi} \times 10^{-1} \text{ dyne cm.}$$

And in general for a machine with "p" pair of poles

$$\text{"}T\text{"} = \frac{p Z \Phi I}{\pi} \times 10^{-1} \text{ dyne cm.}$$

$$= \frac{p Z \Phi I}{4.26 \times 10^{8}} \text{ lb. ft. starting torque}$$

Multiplying by the angular velocity "w", where "w" $= \frac{2\pi N}{60}$, where N equals R.P.M., and dividing by 550 gives power in H.P.

$$\text{"}P\text{"} = \frac{T\, 2\pi N}{60 \times 550} = \frac{p Z \Phi I}{4.26 \times 10^{8}} \times \frac{2\pi N}{60 \times 550}$$

$$P = .000191 \; N \; T \text{ horse power}$$

$$= .1425 \; N \; T \text{ watts}$$

Changing N to R.P.S. and T to # inches

$$P = .71 \; N \; T \text{ watts}$$

Also

$$\text{"}P\text{"} = \frac{p Z \Phi I}{4.26 \times 10^{8}} \times \frac{2\pi N}{550} \times 746 \text{ watts, where N = R.P.S.}$$

$$\frac{2 p Z}{10^{8}} = \text{a constant called } Z'$$

$$\text{"}P\text{"} = \frac{Z' \Phi I N}{4.26} \times \frac{\pi\, 746}{550} = \Phi Z' N I \text{ watts}$$

Equating both values of P

$$.71\,NT = \Phi Z' N I$$

$$"T" = 1.41\,\Phi Z' I \quad \text{lb. inches} \quad \text{starting torque}$$

Running torque equals

$$"T" = \frac{1.41\,P}{N} \quad \text{lb. inches}$$

The impressed electromotive force "E_x" opposes a counter electromotive force "E_c" generated within the motor which equals "$\Phi Z' N$" and forces a current "I_a" through the armature of resistance "R_a", therefore

$$"E_x" = \Phi Z' N + I_a R_a$$

This gives directly expressions for speed and current

$$"N" = \frac{E_x - I_a R_a}{\Phi Z'}$$

$$\text{and } "I_a" = \frac{E_x - \Phi Z' N}{R_a}$$

The power input to the armature of motor therefore equals

$$"E_x I_a" = \Phi Z' N I_a + I_a^2 R_a$$

which does not include friction, windage, hysteresis, and eddy current losses, etc.

The above are the fundamental formulae for all direct current motors.

Since the motor consists essentially of two electrical circuits and a magnetic circuit, it is evident that their

relation to each other can be varied with different connections; i.e., the field circuit can be shunted across the armature circuit, or be connected in series with it, or various combinations of both. This divides the direct current motor into three distinct divisions called the shunt motor, series motor, and the compound motor, respectively. They are all governed by the same fundamental laws, yet are widely different in their behavior, and so will be considered under separate chapters.

CHAPTER II

Shunt D. C. Motor

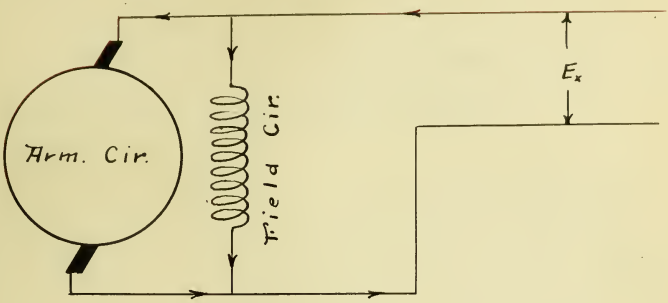


Diagram #5

The above diagram is a simple representation of a shunt motor. The chief characteristic of a shunt motor is the fact that for loads within its working range, the speed is practically constant, not falling more than a few percent. The reason for this is as follows. As long as the field resistance remains constant, and assuming that the impressed voltage remains constant, it is evident that the field current remains constant. Since the field turns are fixed, the magnetio motive force or ampere turns remains constant, and since the magnetic reluctance is constant, then it is evident that by neglecting the armature reactions, that the flux can be considered constant for all loads. From our speed equation

$$N = \frac{E_x - I_a R_a}{\Phi Z'}$$

The change in speed will be due to the change in "I_a" for different loads, but since "R_a" is very small, the "$I_a R_a$" drop is

very small.

Considering then that "N" is practically constant, from the equation

$$P = .71 \; N \; T$$

it is evident that "P" varies directly as the torque. Also from the equation

$$"T" = 1.41 \Phi Z' I_a$$

it is evident that in a shunt motor the torque varies directly as the current, and will be a straight line. The limiting value of current, and therefore of torque, will be due to heating. It is also very plain from the equation

$$"T" = 1.41 \Phi Z' I_a$$

that the same value of torque can be produced by changing the relative values of "Φ", "Z'", and "I" over a considerable range. This means that by increasing the flux "Φ" and the factor "Z'", "I" can be decreased and the torque "T" still remain constant, so that the specific torque or torque per ampere is increased. This can best be shown by considering an actual case where $\Phi = 5 \times 10^6$ $Z' = 6 \times 10^{-6}$, and the relation between "T" and "I " is as tabulated.

"I"	0	10	25	50	75	100	125	150
"T "	0	423	1058	2115	3170	4230	5300	6350

By increasing "Φ" to 10×10^6 lines, and Z' to 7×10^{-6} the following relation is obtained as tabulated.

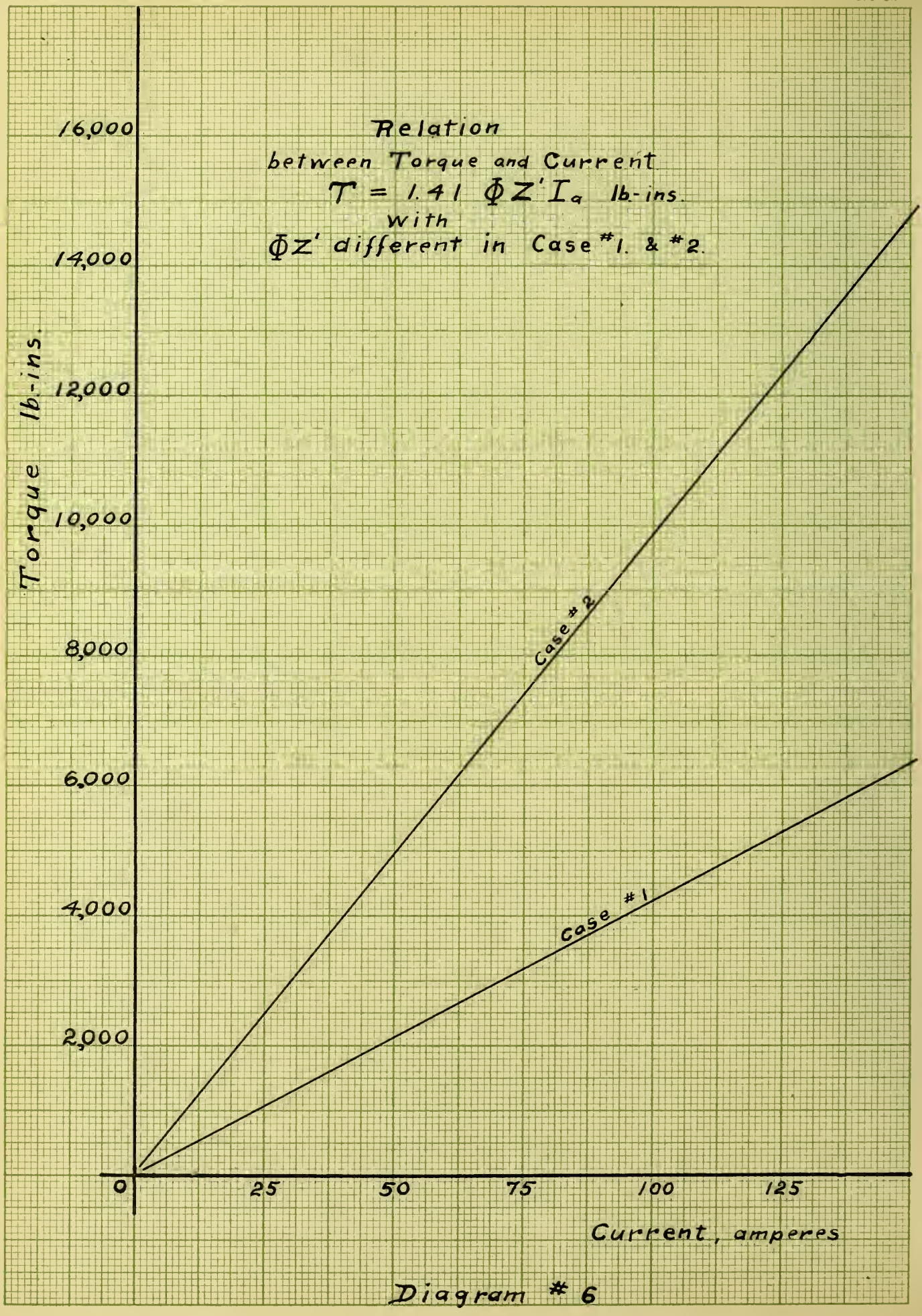

Relation
between Torque and Current
$T = 1.41\ \Phi Z' I_a$ lb-ins.
with
$\Phi Z'$ different in Case #1. & #2.
16,000
14,000
12,000
10,000
8,000
6,000
4,000
2,000
Torque lb-ins.
Case #2
Case #1
0
25
50
75
100
125
Current, amperes

Diagram # 6

I	0	10	25	50	75	100	125	150
T	0	986	2465	4930	7400	9860	12300	14780

The results are best shown by graphs, and diagram #6, page 14, shows the relations existing in the two cases. In case 1, the specific torque in # ft per ampere is 35.2, while in case 2, the specific torque is 82.2.

This introduces the very important consideration of losses. The important losses in a motor are the iron losses, consisting of hysteresis and eddy current losses; the friction and windage loss, which is entirely mechanical; and the copper losses, composed of field and armature I^2R losses. Of these, the armature I^2R loss is a variable loss and all the rest are constant losses for a shunt machine. So that the most important consideration in deciding, in the design of a motor as to the value of specific torque to use, will involve a study of the losses that will be incurred. For example, a large specific torque will mean a large flux or large number of conductors; this will mean large iron losses, and as the current necessary will be small, the armature I^2R loss will be small. It is evident that almost any combination could be used, but such things as the cost of manufacture and the nature of service need be considered. It is found that in machines of the same size and of equally good design, that the average losses through the working range are about equal, so that if a machine has a high efficiency at light loads, it has it at the expense of its full load efficiency, and vice versa.

The starting current will depend upon the current necessary to bring the machine up to normal speed under a definite load. While the machine is at rest, no counter e.m.f. is developed, so that if "E" were impressed the current would be limited only by "R" and would be immense; and would ruin the machine. It is usually limited to 150% or 200% of normal full load value by a starting resistance in series with the armature, which reduces the pressure across the armature. This resistance is gradually cut out as the counter electromotive force takes the place of the resistance drop.

To better consider the characteristics of a shunt machine, take for example a 100 B.H.P. motor rated to run at 1,000 R.P.M. under full load, on a 500 volt circuit. Assume that at full load its efficiency is 90%, and that at full load its constant and variable losses are 60% and 40% of total losses, respectively. By applying the formulae derived for the direct current motor under the assumed conditions, the following values are obtained:

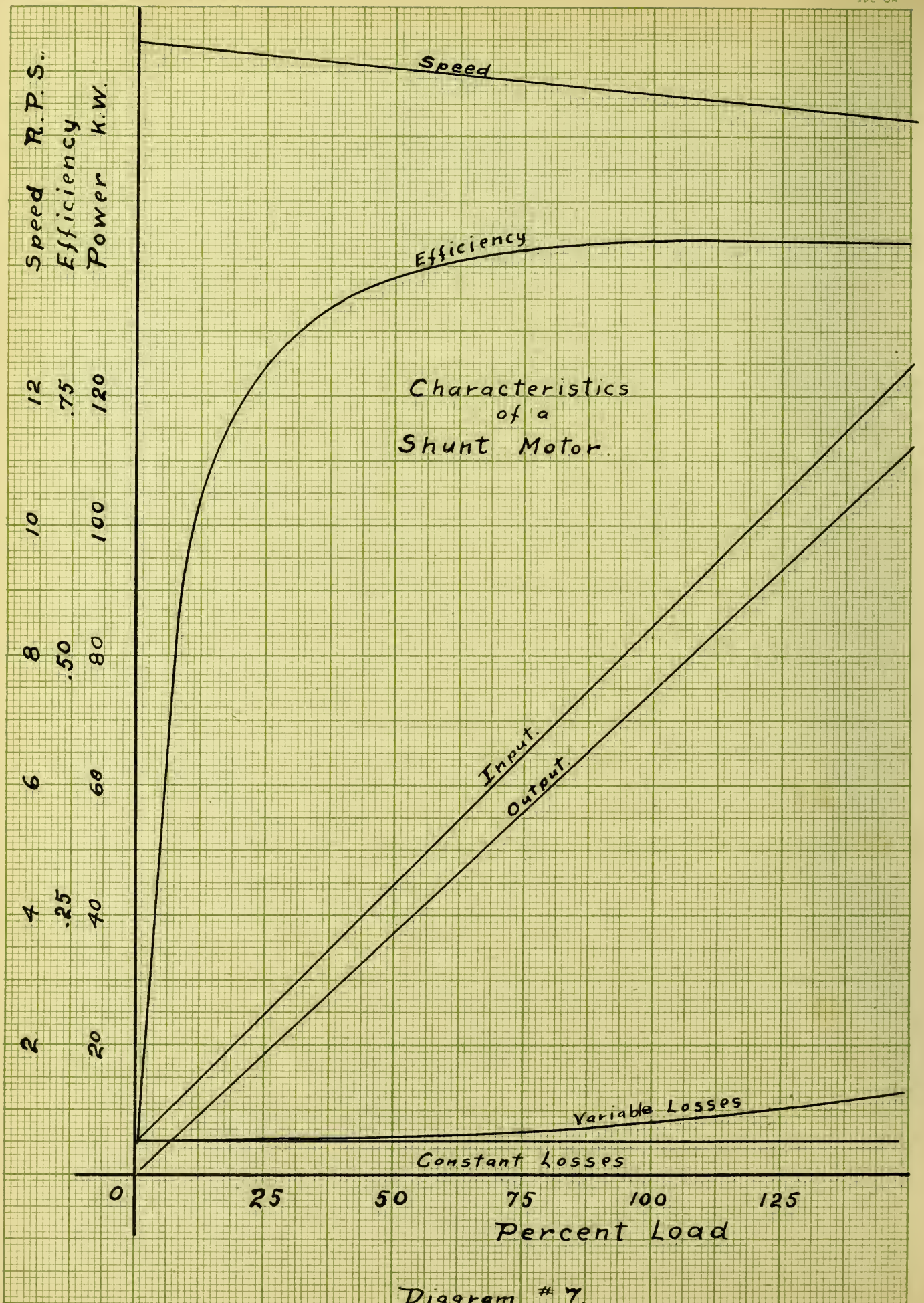
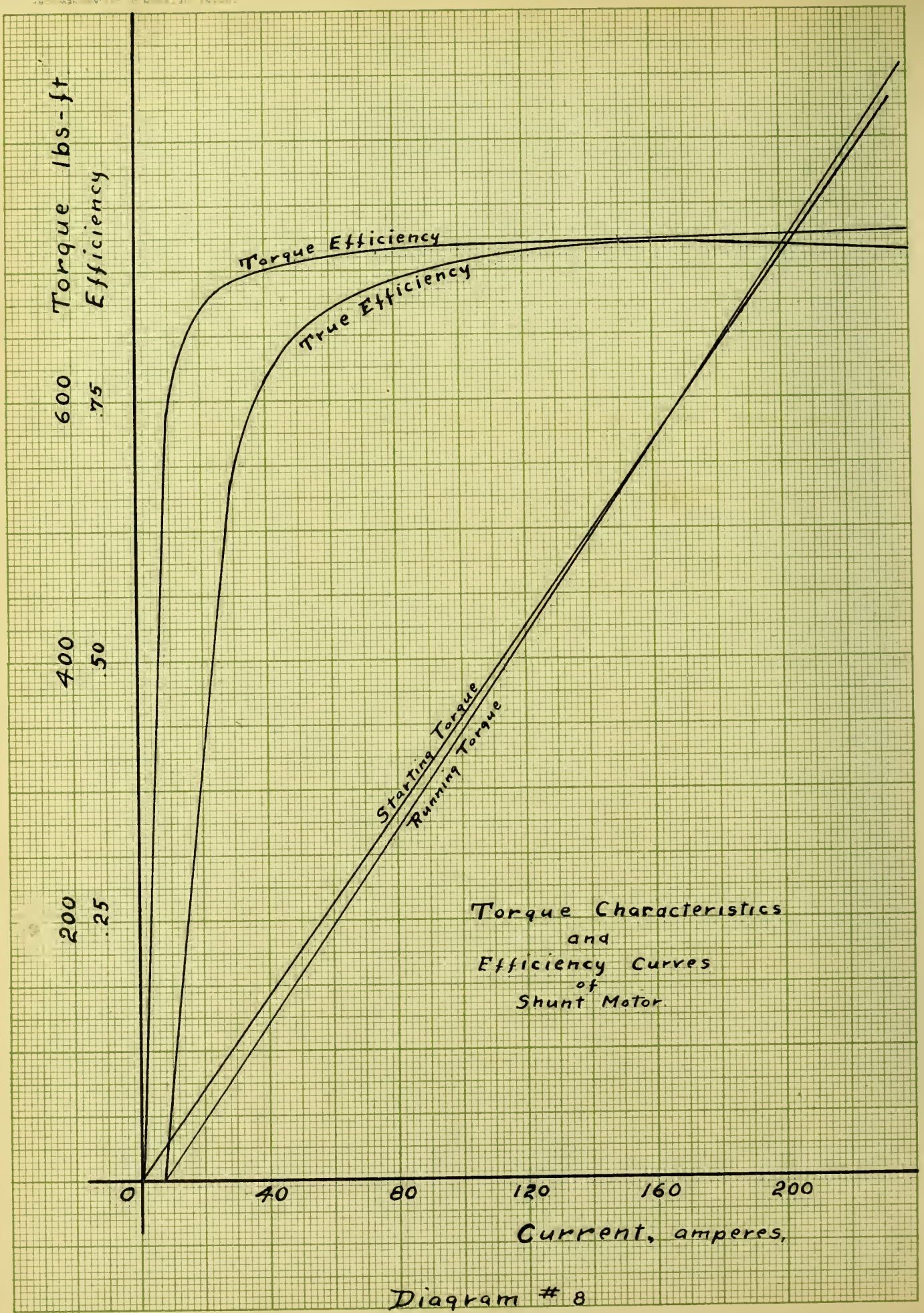

Speed R.P.S.
Efficiency
Power K.W.
Speed
Efficiency
Characteristics
of a
Shunt Motor.
Input.
Output.
Variable Losses
Constant Losses
12
.75
120
10
100
8
.50
80
6
60
4
.25
40
2
20
0
25
50
75
100
125
Percent Load

Diagram # 7.

Percent Load	0	25.8	51.	75.7	100	123.3	146.
Output H.P.	0	25.8	51	75.7	100	123.3	146
Efficiency	0	.785	.867	.89	.90	.898	.897
Input K.W.	5.04	24.49	43.94	63.44	83.	102.34	121.54
Losses(total)KW	5.04	5.25	5.88	6.935	8.4	10.3	12.59
" (I R) "	0	.21	.844	1.895	3.36	5.26	7.55
" (Constant)KW	5.04	5.04	5.04	5.04	5.04	5.04	5.04
" (field I R) "	.84	.84	.84	.84	.84	.84	.84
" (Const.Arm) "	4.2	4.2	4.2	4.2	4.2	4.2	4.2
Input to Armature"	4.2	23.65	43.1	62.6	82.16	101.5	120.7
Ex	550	550	550	550	550	550	550
I (total)	7.64	43.04	78.44	113.84	149.1	184.64	219.64
I (light)	7.64	7.64	7.64	7.64	7.64	7.64	7.64
I (to overcome R)	0	35.4	70.8	106.2	141.46	177.	212.
R_a	.168	.168	.168	.168	.168	.168	.168
$\Phi Z'$	31.4	31.4	31.4	31.4	31.4	31.4	31.4
$\Phi Z'$(effective)	30	30	30	30	30	30	30
Number of Poles	4	4	4	4	4	4	4
Z	125	125	125	125	125	125	125
Z'	$5x10^{-6}$	$5x10^{-6}$	$5x10^{-6}$	$5x10^{-6}$	$5x10^{-6}$	$5x10^{-6}$	$5x10^{-6}$
Φ	$6x10^{6}$	$6x10^{6}$	$6x10^{6}$	$6x10^{6}$	$6x10^{6}$	$6x10^{6}$	$6x10^{6}$
$I_a R_a$	1.29	7.23	13.2	19.1	25.1	31.	36.8
Speed R.R.S.	17.45	17.27	17.08	16.9	16.66	16.5	16.3
Torque lbs.ft.	0	130.8	261	392	525	654	785



Torque Characteristics
and
Efficiency Curves
of
Shunt Motor

Diagram # 8

Plotting speed, input, output, losses, and efficiency as ordinates, and percent load as abscissa, some of the characteristics of the shunt motor are obtained as shown on diagram #7, page 17.

The starting torque of the motor under consideration can be obtained by assuming various starting currents supplied. The results are tabulated as follows:

Current	0	25	50	75	100	150	200
Starting Torque #ft.	0	88.2	176.4	264.6	352.8	530	706
$T_s \times S_n \times 2\pi \frac{746}{550}$	0	12.5	25.1	37.6	50.2	75.3	100.4
Input	.84	14.59	28.34	42.09	55.84	83.34	110.84
Torque Efficiency	0	.857	.885	.892	.90	.904	.907

Plotting running torque from table on page 18, starting torque, true efficiency from table on page 18, and torque efficiency as ordinates, against current as abscissa, the relation between running and starting torque, and the relation between true efficiency and torque efficiency are readily seen. These characteristics are shown on diagram #8, page 19.

It is seen that torque efficiency is much higher at light loads than true efficiency, but that at full load they are equal. It is also noted that if a value of current equal to full load current were forced through the motor that the running torque at full load is equal to the starting torque. By plotting speed as ordinate, and torque as abscissa, it is advantageous to draw in lines of constant horse power as shown on diagram #9, page 21. From these lines, for any value of

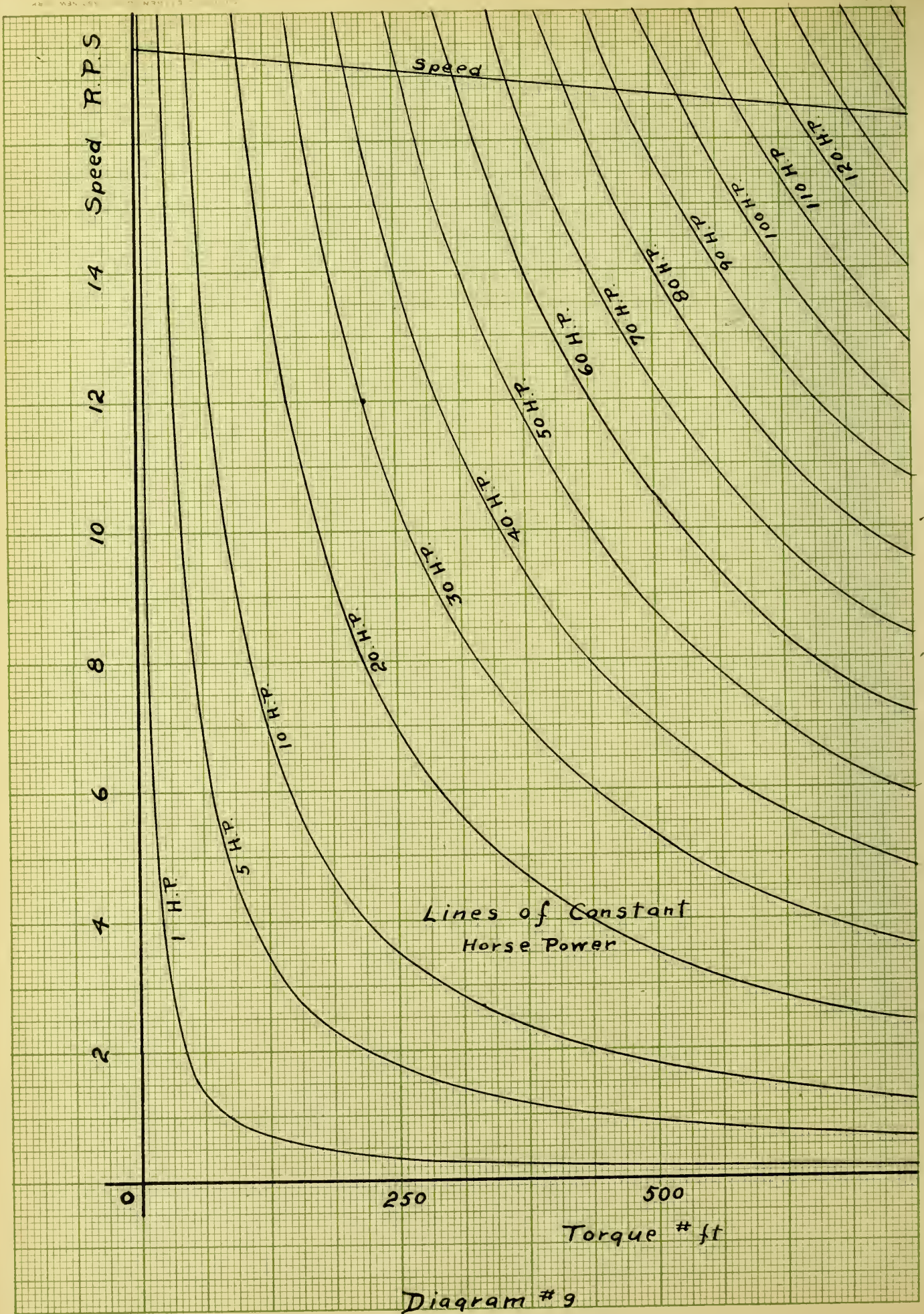

Speed R.P.S
14
12
10
8
6
4
2
0
250
500
Torque # ft
Speed
120 H.P.
110 H.P.
100 H.P.
90 H.P.
80 H.P.
70 H.P.
60 H.P.
50 H.P.
40 H.P.
30 H.P.
20 H.P.
10 H.P.
5 H.P.
1 H.P.
Lines of Constant
Horse Power

Diagram # 9

horse power desired, it is possible to select any speed and the required torque, within the limits of speed and torque. The speed curve of the motor considered previously shows the speed at different power outputs.

The shunt motor has been found in this chapter to be essentially a motor of practically constant speed, making it desirable for driving line shafting in factories, and all machines where a constant speed for all loads is desired. Its speed can be changed by different methods when desired, but that is not within the scope of this treatise. The torque varies directly as the load and as the current, so that the torque that a shunt motor will give is limited only by the value of current where heating is disastrous to the machine.

CHAPTER III

Series D. C. Motor

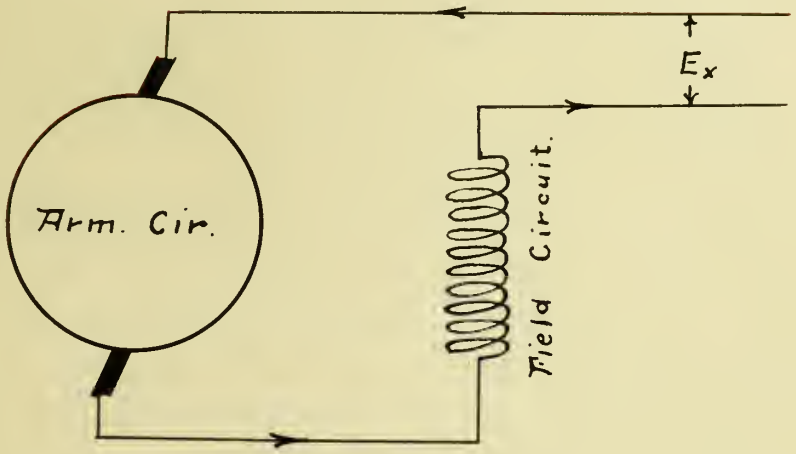


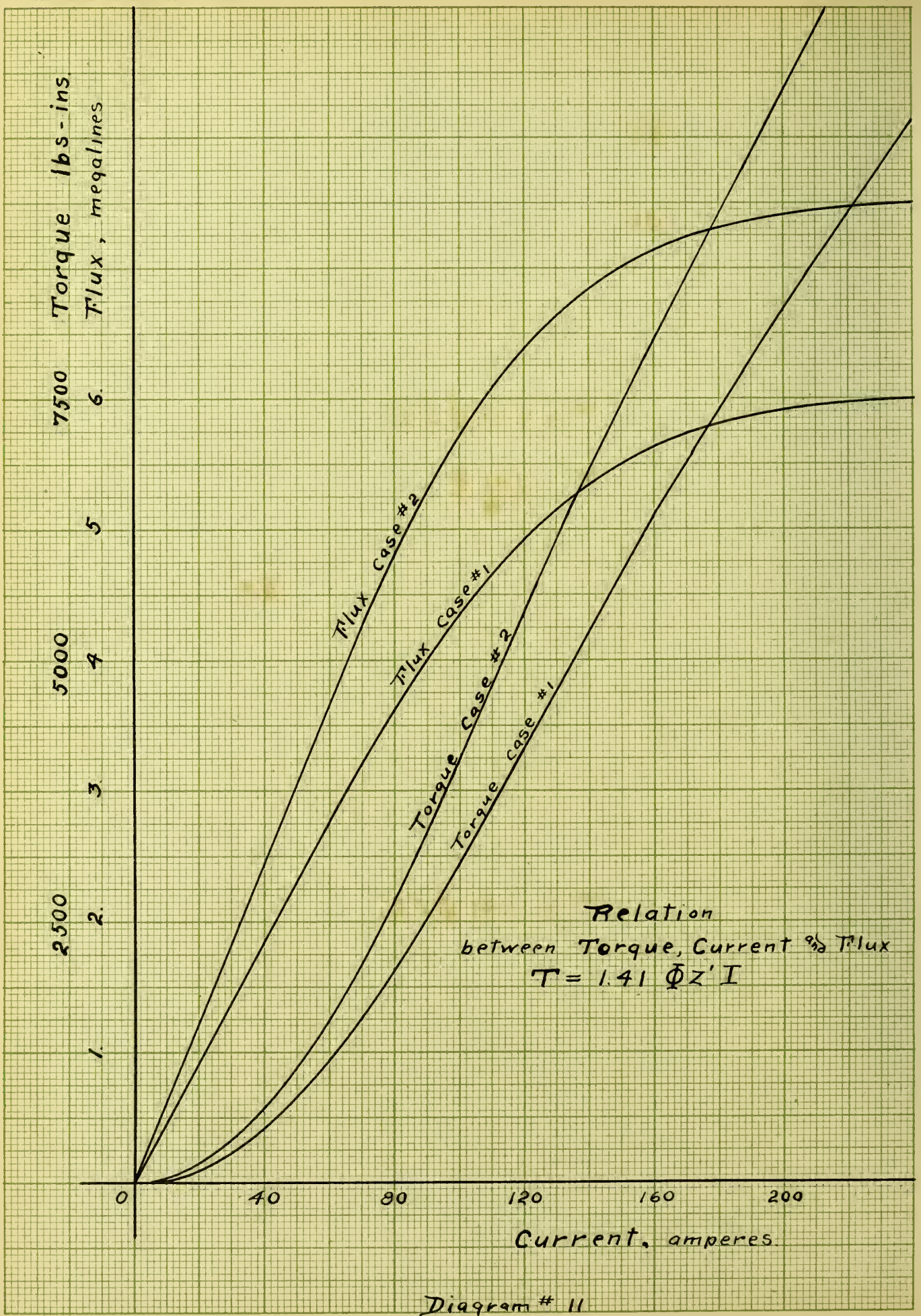
Diagram #10

The above is a simple diagram of a series motor, so called because its field circuit is in series with the armature circuit. It is evident, with this arrangement, that the field flux, since it depends upon the field current, will vary for different loads, because the field current and armature current are the same thing. The total current, therefore, depends upon the counter electromotive force, the field resistance, and the armature resistance, so that

$$"I" = \frac{E_x - \Phi Z' N}{R_a + R_f}$$

The speed of a series motor is also a very important consideration, and is expressed by

$$"N" = \frac{E_x - I(R_a + R_f)}{\Phi Z'}$$



Relation
between Torque, Current & Flux
$T = 1.41 \Phi z' I$

Diagram # 11

Theoretically "$\Phi Z'$" will be infinitely small at no load, so that the speed becomes infinitely great. As load is increased "I_a" increases and "$\Phi Z'$" increases, so that with increased load "N" decreases very rapidly. It is for this reason that a series motor will race at light loads.

The torque equations of the direct current motors

$$"T" = 1.41\ \Phi Z' I_a$$

and $$"T" = \frac{1.41\ P}{N}$$

but they have a very different significance in a series motor than in a shunt motor, due to the fact that "$\Phi Z'$" increases with "I_a" in a manner dependent upon the saturation curve of the magnetic circuit. This can best be considered by taking an example. Assume a relation between flux and current as is shown by case #1, diagram #11, page 24; assume a value of 5×10^{-6} for "Z", and for various values of current "I_a" the value of torque can be calculated from

$$"T" = 1.41\ \Phi Z' I_a$$

as is tabulated below.

Case #1

Current	0	40	80	120	160	200	240
Z'	5×10^{-6}	5×10^{-6}	5×10^{-6}	5×10^{-6}	5×10^{-6}	5×10^{-6}	5×10^{-6}
Φ	0	1.85×10^{6}	3.62×10^{6}	4.92×10^{6}	5.63×10^{6}	5.91×10^{6}	6×10^{6}
$\Phi Z'$	0	9.25	18.2	24.6	28.15	29.6	30
"T"	0	522	2050	4170	6400	8350	10150
lbs/amp.	0	1.085	2.14	2.89	3.33	3.47	3.52

The effect of changing the relation between $\Phi Z'$ and "I," may be shown by considering another example, assuming a relation between current and flux as shown on diagram #11, page 24, as case #2. Results are as follows:

Current	0	40	80	120	160	200	240
Z'	5×10^{-6}	5×10^{-6}	5×10^{-6}	5×10^{-6}	5×10^{-6}	5×10^{-6}	5×10^{-6}
Φ	0	2.45×10^{6}	4.8×10^{6}	6.38×10^{6}	7.15×10^{6}	7.4×10^{6}	7.5×10^{6}
$\Phi Z'$	0	12.25	24	31.9	35.7	37.	37.5
"T"	0	692	2710	5400	8060	10450	12700
ft.lbs/amp	0	1.41	2.82	3.75	4.19	4.35	4.4

A study of the results and curves shows that an increase of flux with current brings an increase of specific torque.

The chief characteristic of a series motor is the fact that at large loads it has a large starting torque. This is due to the fact that at large loads the current is

necessarily large, and the flux also, since it depends upon current, is large, so that from the equation

$$T = 1.41\,\Phi Z' I_a$$

"T" must also be very large. The large torque in a series motor is obtained at the expense of speed, for speed drops off very rapidly with increase of load. The motor will speed up to a value such that the output plus losses equals the input. Just as in a shunt motor, so also in a series motor, the torque can be produced by different combinations of flux, current, and number of conductors. The principal differences in the different designs will be in the division of losses, and the shapes of efficiency curves.

Losses in a series motor are the iron losses, friction, and windage, field "$I_f^2 R_f$" and armature "$I_a^2 R_a$". The variable losses in the series motor includes both the field and armature "I^2R" losses, so that constant losses are chiefly iron losses and friction and windage. Theoretically the iron loss is not constant, as the flux varies from almost zero to complete saturation of the magnetic circuit. But iron loss also depends upon frequency of hysteresis and eddy currents, so that speed is an important factor. The result is that speed decreases as the flux increases, and the iron losses can be considered, for this discussion, as practically constant. The characteristics of a series motor can best be studied by considering an example. Assume a 100 B.H.P. series motor, designed to give maximum efficiency of 90% at one-half load, and at that load to give a

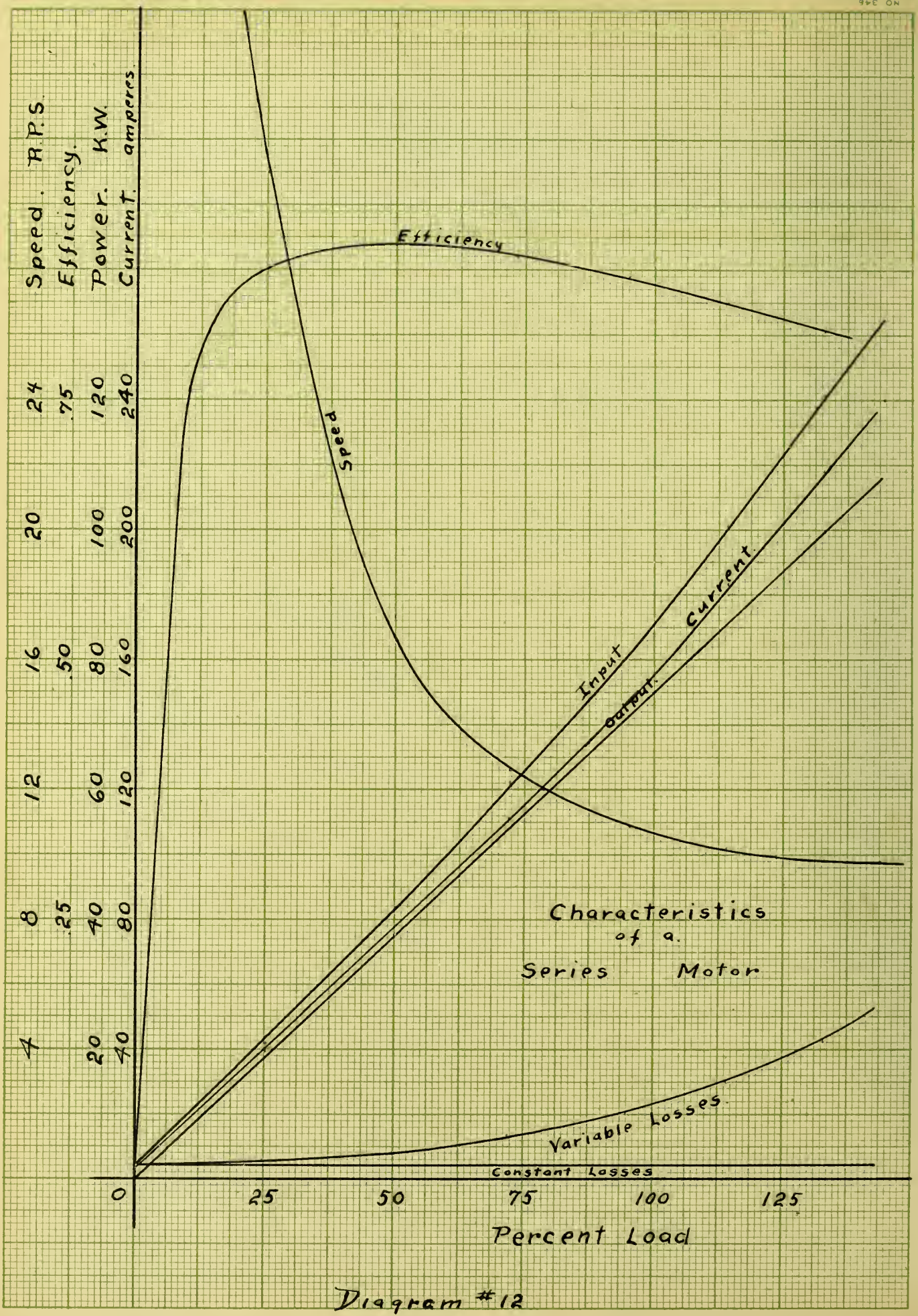

Speed. R.P.S.
Efficiency.
Power. K.W.
Current. amperes.
24
.75
120
240
20
100
200
16
.50
80
160
12
60
120
8
.25
40
80
4
20
40
Efficiency
Speed
Input
Output.
Current.
Characteristics
of a.
Series Motor
Variable Losses
Constant Losses
0
25
50
75
100
125
Percent Load

Diagram #12

speed of 1,000 R.P.M. The machine is to operate upon a 550 volt circuit. The results are tabulated as follows:

Percent Load	0	25.7	50	73.	94.5	114	133
Output H.P.	0	25.7	50	73	94.5	114	133
Efficiency	0	.88	.90	.89	.87	.847	822
Input K.W.	2.1	21.8	41.5	61.2	81.	100.5	120.5
Losses (total)	2.1	2.63	4.2	6.73	10.5	15.3	21.1
Losses (I^2R)	0	.53	2.1	4.63	8.4	13.2	19
Losses (Const)	2.1	2.1	2.1	2.1	2.1	2.1	2.1
Ex	550	550	550	550	550	550	550
I (total)	3.82	39.66	75.50	111.32	147.12	182.82	218.82
I (light)	3.82	3.82	3.82	3.82	3.82	3.82	3.82
I (over-come R)	0	35.84	71.68	107.5	143.3	179	215
$R_a + R_f$	.41	.41	.41	.41	.41	.41	.41
$\Phi \times 10^6$	.2	2.2	3.95	5.1	5.7	5.93	6
$Z' \times 10^{-6}$	7.85	7.85	7.85	7.85	7.85	7.85	7.85
$\Phi Z'$	1.57	17.25	31.	40	44.75	46.5	47
Z	196	196	196	196	196	196	196
# poles	4	4	4	4	4	4	4
$I_a R_{a+f}$	1.57	16.3	31	45.7	60.5	75	89.6
R.P.S.	∞	31	16.66	12.6	10.9	10.2	9.8
Torque #ft	0	72.5	252	506	760	980	1185

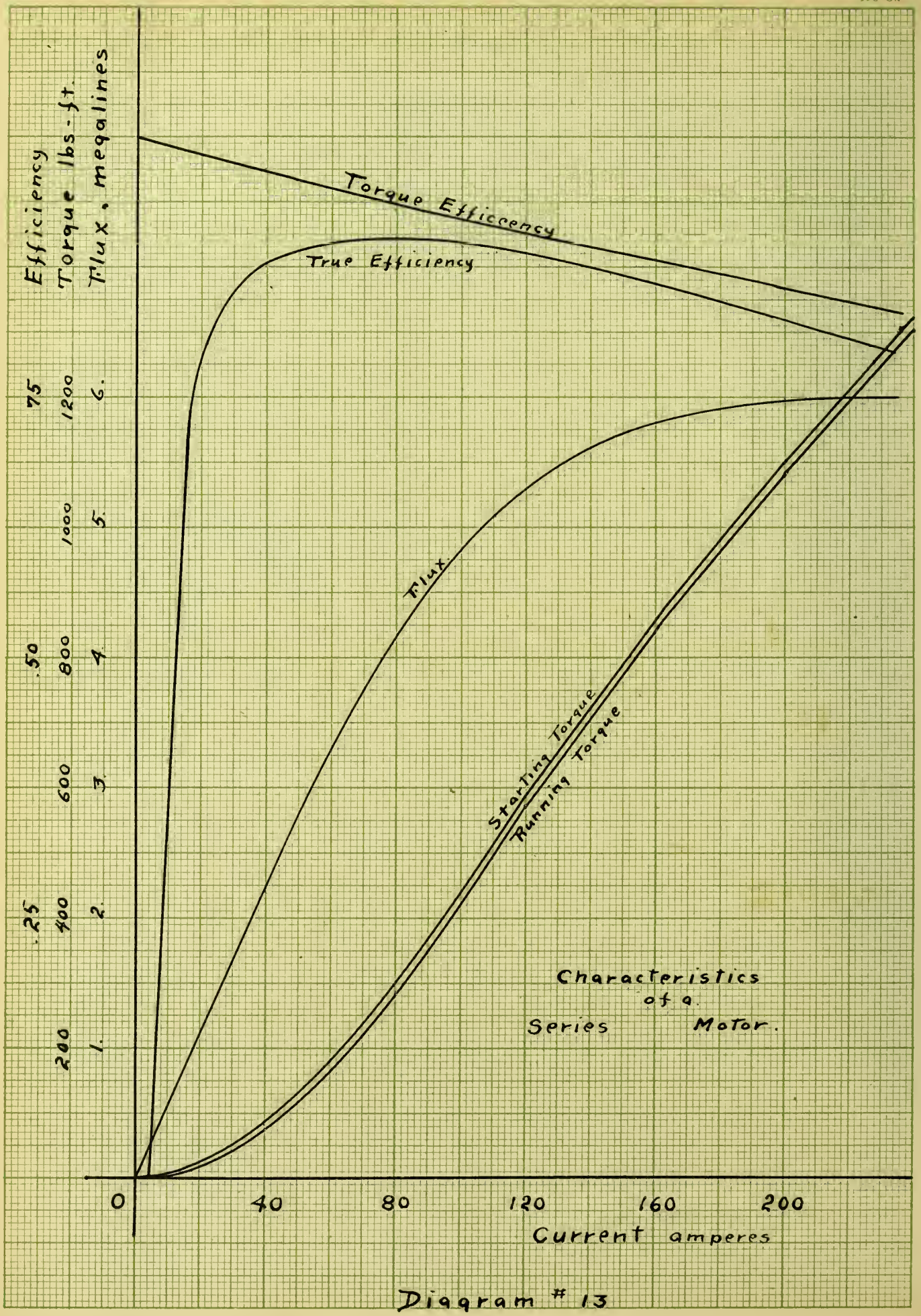

Efficiency
Torque lbs-ft.
Flux. megalines
Torque Efficiency
True Efficiency
Flux
Starting Torque
Running Torque
.75
1200
6.
1000
5
.50
800
4
600
3
.25
400
2
200
1
0
40
80
120
160
200
Current amperes
Characteristics
of a
Series Motor.

Diagram # 13

Plotting speed, efficiency, input, output, current, and losses as ordinates, against percent load as abscissa, some of the characteristics of the series motor are obtained. They are shown on diagram #12, page 28.

The starting torque of the series motor under consideration can be obtained for different values of current forced through the machine, since the relation between flux set up and current is known from the above tabulation. The values of starting torque are therefore calculated and tabulated as follows:

Current	0	40	80	120	160	200	240
Flux Φ	0	2.2×10^{6}	4.13×10^{6}	5.3×10^{6}	5.8×10^{6}	5.95×10^{6}	6×10^{6}
Z'	7.85×10^{-6}	7.85×10^{-6}	7.85×10^{-6}	7.85×10^{-6}	7.85×10^{-6}	7.85×10^{-6}	7.85×10^{-6}
$\Phi Z'$	0	17.25	32.4	41.6	45.5	46.7	47.
Starting torque #ft	0	81.2	296	587	857	1100	1325
Speed R.P.S.	∞	31	16.2	12	10.6	10	9.8
$T_s \times S_n \times 2\pi \frac{746}{550}$	0	21.5	41	60	77.6	94	111
Input	0	22.	44	66	88	110	132
Torque Efficiency	0	.978	.932	.91	.882	.854	.84

Plotting flux, starting torque, running torque, true efficiency, and torque efficiency, as ordinates, against current as abscissa, the relation between starting and running torque, and the relation between true efficiency and torque

efficiency are readily seen. Refer to diagram #13, page 30.

The fact that the series motor can exert such a large starting torque on heavy loads makes it of great importance on such classes of service where large starting torques are required, and speed regulation is not important. For this reason, the series motor is used on railway cars and cranes and that class of service. Series motors are rated at the maximum torque they will exert, but can not be operated on more than half load continuously, as heating would be disastrous to the machine.

CHAPTER IV

Compound D. C. Motor

The compound d. c. motor combines the shunt and the series wound motors, so that the motor has both shunt and series field windings. It is evident that these windings may be so wound that they will aid each other, or on the other hand can be wound to oppose each other. We therefore have two kinds of compound motors, the cumulative and the differential.

In the cumulative wound motor, an increase of load increases the series field current, and the series flux increases, increasing the shunt field flux, and consequently the total flux.

This increase in flux we see from our speed equation

$$"N" = \frac{E_x - I_a(R_a + R_f)}{\Phi Z'}$$ causes a decrease in speed that depends upon the relative proportion between the series and the shunt field magneto-motive forces.

The motor has, however, the advantage of large starting torque, as is evident from the equation

$$"T" = 1.41\ \Phi Z' I_a$$

The cumulative compound motor therefore combines the shunt and series d.c. motors, and gives a motor that will not race, has a large starting torque, and a poor speed regulation.

In the differential compound motor, the series winding opposes the shunt wind, so that an increase of load decreases

the flux and the motor speeds up. The starting torque is decreased. These changes are proportional to the series and shunt magneto-motive forces. The differential compound motor is usually designed to give perfect speed regulation, so that the amount of series flux opposing the shunt flux is just enough to keep the motor speed constant.

CHAPTER V

Polyphase Induction Motor

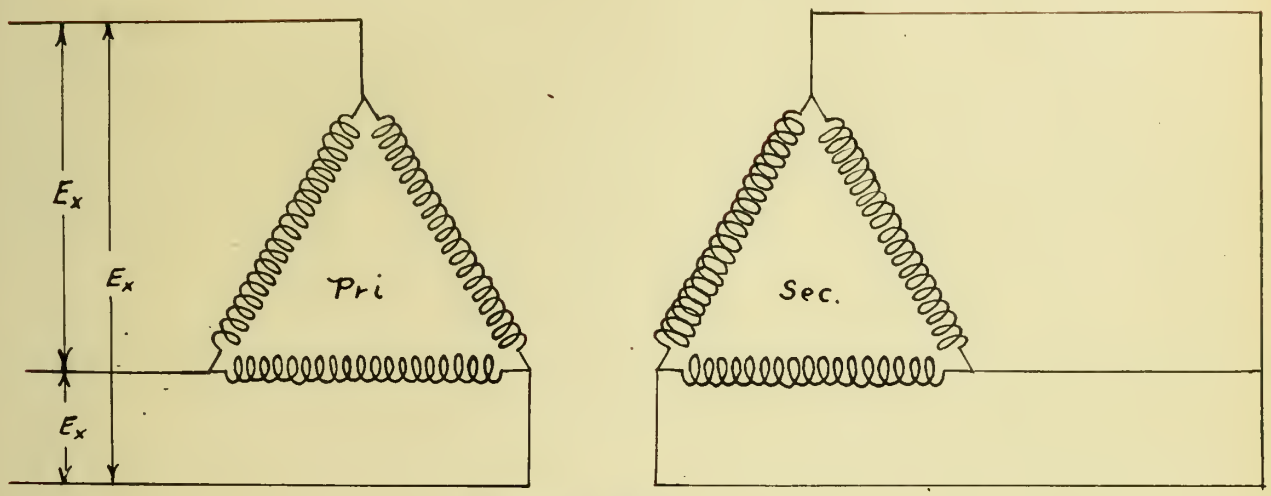


Diagram #14

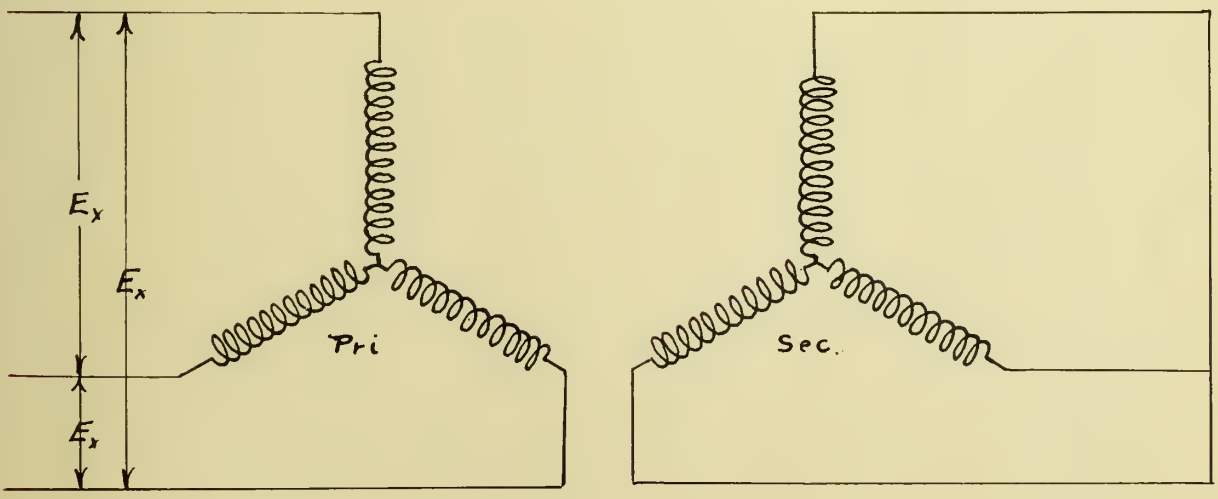


Diagram #15

Either of the above diagrams represent in simplest form a three phase induction motor, according as it is Δ or Y wound. The principle of the induction motor is in general that of the transformer except that the secondary circuit, in

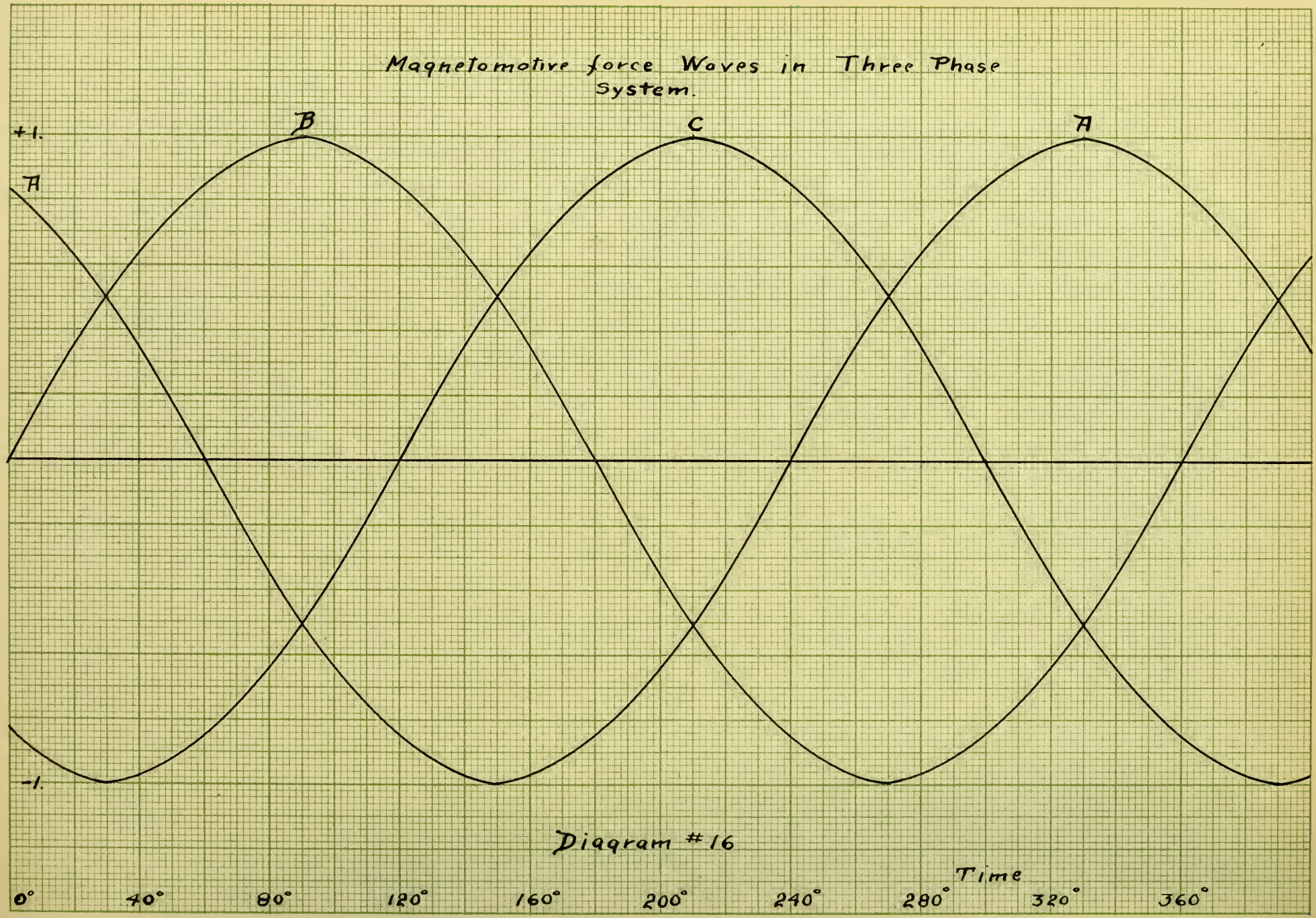


Diagram #16

NO 346

the case of the induction motor, is free to rotate about an axis. If the induction motor were blocked, it could be used as a transformer. There are in general two types of induction motors: the squirrel cage, and the wound rotor. The difference in them is that in the squirrel cage motor the secondary conductors are copper bars in armature slots, all connected electrically to a copper ring on each end of the armature, while the wound rotor has the ordinary windings. The nature of the primary winding in both, and the secondary of the wound rotor, is the same as is found in a polyphase alternator.

As was before stated, the principle of the induction motor is that of the transformer, except that the secondary is free to rotate. The magnetic path is still present in the motor, for the flux is forced across an air gap that is made as small as possible. The flux, in streaming across the air gap, cuts across the conductors of the rotor; an electromotive force is induced in them that forces a current through the conductors that is limited by the value of resistance in the secondary. This induced current sets up a flux about the conductors, and a torque is produced, as explained in the introduction, and the rotor revolves. The main flux, which is set up by the primary, is in a state of rotation, so that it really tends to drag the rotor conductors around at as near the same speed as possible.

The fact that the main flux rotates may be shown as follows. In diagram #16, page 36, curves A, B, and C represent

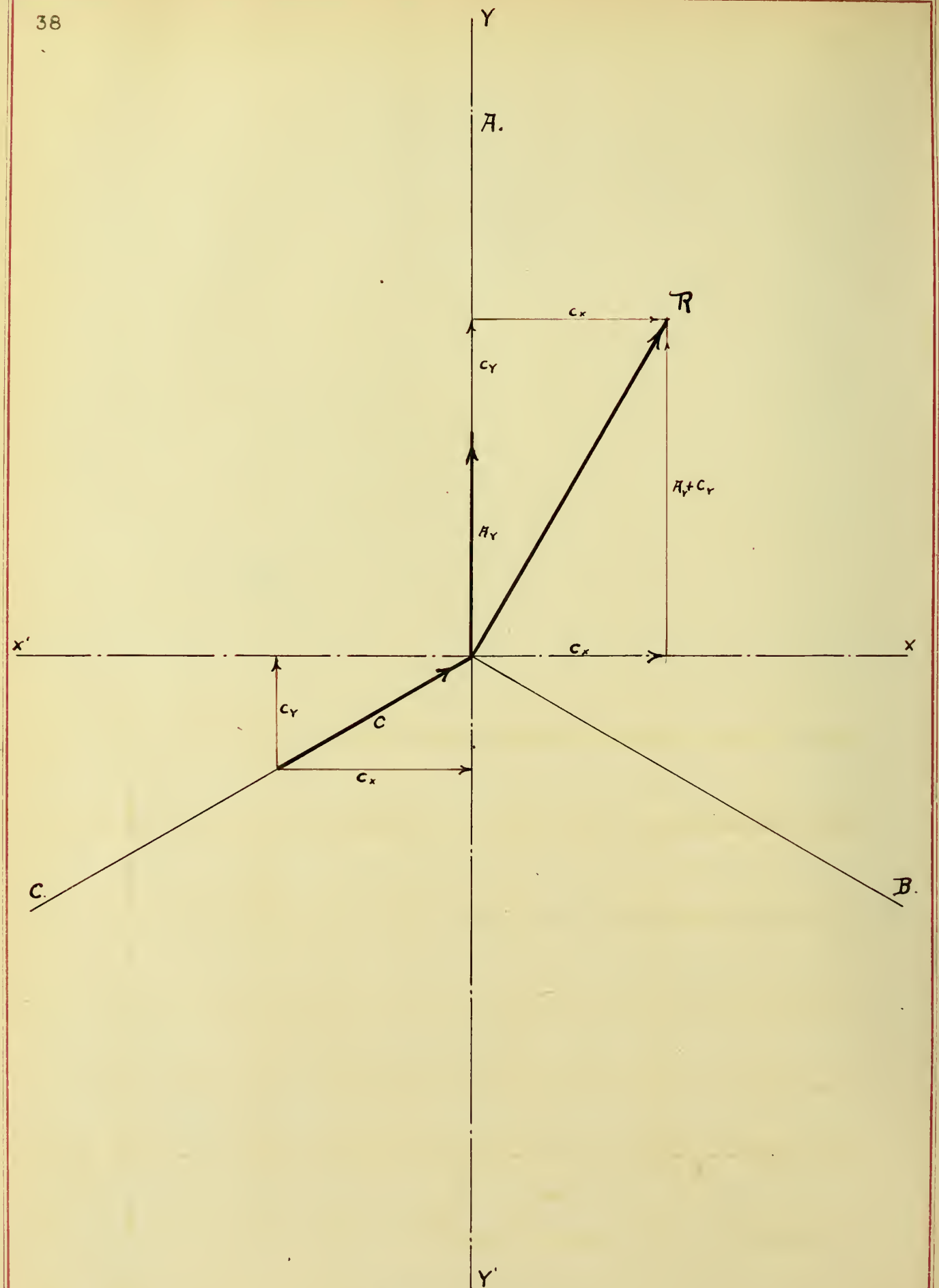
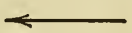


Diagram # 17

the waves of magneto-motive force in the three coils of primary winding at 120° phase displacement. The resultant flux at consecutive instances may be obtained in percent of the maximum of the separate waves. Take, for example, the instant 0°. It is seen that the value of A = .866, B = 0, and C = .866. If these values be drawn as vectors at 120° displacement, as shown in diagram #17, page 38, and their horizontal and vertical components be summed up, the resultant components will be the components of the resultant flux at that instant, showing both magnetitude and direction, if drawn to scale. The same may be done for all instances.

The following table shows results for a succession of instances at equal intervals representing one complete cycle.

Time	0	60	120	180	240	300
Ay	.866	0	-.866	-.866	0	.866
Ax	0	0	0	0	0	0
By	0	-.433	-.433	0	.433	.433
Bx	0	.75	.75	0	-.75	-.75
Cy	.433	.433	0	-.433	-.433	0
Cx	.75	.75	0	-.75	-.75	0
Ry	1.299	.0	-1.299	-1.299	0	1.299
Rx	.75	1.5	.75	-.75	-1.5	-.75
R	1.5	1.5	1.5	1.5	1.5	1.5
Direction						

The above results show that the value of flux is constant in value, and that its direction is constantly changing. The resultant flux makes one complete revolution per cycle of component fluxes, and also per cycle of voltage.

The synchronous speed of the induction motor is therefore the speed of the magnetism, which depends upon the frequency of the impressed voltage and the number of polar regions of the motor.

$$N = \frac{2f}{\#\ poles}$$

The number of polar regions in the motor equals the number of bands of stator conductors per phase of stator windings.

The motor will never run at exactly synchronous speed, because in that case the rotor conductors would have to follow exactly with the speed of the magnetism, and if they did there would be no cutting of lines of flux, consequently no voltage induced, and therefore no current to produce the torque necessary. Exact synchronous speed would be reached only under the ideal condition of no load with no losses and no resistance in the rotor conductors.

If the induction motor is required to run under a heavy load, the torque necessary will be large, and the current required will also be large. In order that a large current flow against rotor resistance, a large induced voltage must be set up. This is obtained by the cutting of the flux at a greater speed, so that in order that this follows, the rotor must slip behind the speed of the magnetism sufficiently to

give the necessary induced voltage. The amount that the rotor will slip in this case is dependent upon the rotor resistance, as the amount of resistance determines the necessary electromotive force for any value of current. There is not an exact proportion between the speed and the torque, because the increase of current demagnetizes the flux. The decrease in torque, therefore, is two-fold, because the decrease of flux decreases the induced electromotive force and consequently the current; and the action of a decreased flux upon conductors with decreased current causes a marked decrease in torque. From the above statement it is evident that there must be a value of flux that is maximum for the machine, after which further increase of load will bring the motor to a stand-still. A motor is said to break down after this point has been passed.

The speed at which this maximum torque will occur depends upon the value of rotor resistance, for if at any speed the resistance is increased, the motor must slip a greater amount to furnish the current for the maximum value of torque. The resistance could be increased sufficiently so that the maximum torque will occur at starting.

This fact is made use of by short-circuiting the rotor windings through resistances, and cutting them out again after the machine has started. Resistance means large I^2R losses and consequently poor efficiency, so that resistance is never put in for any but short periods.

Losses in the induction motor are due to hysteresis and eddy currents in the stator and rotor core, to copper

losses in the windings, and to the friction and windage of the rotor.

The characteristics of an induction motor may best be shown by considering a machine, and with the necessary assumptions calculate the necessary relations. To calculate the induction motor, the method of complex numbers is most easily handled.

Let	r_p	equal	primary resistance
"	r_s	"	secondary resistance
"	x_p	"	primary reactance
"	x_s	"	secondary reactance
"	g	"	admittance to coreloss current
"	b	"	admittance to magnetizing current
"	s	"	percentage of slip behind synchronous speed
"	E_x	"	impressed voltage
"	e	"	secondary e.m.f. at stand-still

Then the primary impedance equals $r_p - jx_p$, secondary impedance equals $r_s - j(x_s s)$, and exciting current admittance equals $g + jb$.

The secondary voltage equals "se", therefore secondary current equals

$$\frac{se}{r - j x_s s} = e\left[\frac{sr}{r_s^2 + \overline{sx_s}^2} + j\frac{s^2 x_s}{r_s^2 + \overline{sx_s}^2}\right]$$

$$I_s = e\left[a_1 + j a_2\right]$$

where $a_1 = \frac{sr}{r_s^2 + \overline{sx_s}^2}$ and $a_2 = \frac{s^2 x_s}{r_s^2 + \overline{sx_s}^2}$

The exciting current equals

$$"I_{ex}" = e(g + jb)$$

Therefore primary current equals

$$"I_p" = e\left[(a_1 + g) + j(a_2 + b)\right]$$

$$= e(b_1 + jb_2)$$

where $"b_1" = a_1 + g.$ and $"b_2" = a_2 + b.$

$$"E_x" = e + I_p Z_p = e + e(b_1 + jb_2)(r_p - jx_p)$$

$$= e + e\left[(b_1 r_p + b_2 x_p) + j(b_2 r_p - b_1 x_p)\right]$$

$$= e\left[(1 + b_1 r_p + b_2 x_p) + j(b_2 r_p - b_1 x_p)\right]$$

$$= e(c_1 + jc_2)$$

where $"c_1" = (1 + b_1 r_p + b_2 x_p)$ and $"c_2" = (b_2 r_p - b_1 x_p)$

The watts supplied to the motor by $"I_p"$ and $"E_x"$

equals $"P" = e^2(b_1 c_1 + b_2 c_2)$

The volt amperes supplied equals $"I_p E_x"$

$$\therefore \text{ Power factor } = \frac{e^2(b_1 c_1 + b_2 c_2)}{E_x I_p}$$

The torque in the secondary in synchronous watts

equals $"T\ syn." = e^2 a_1$ where "T syn" would be power in watts if motor ran at syn. speed.

Transforming to lbs.ft.

$$"T_{syn}" = \frac{2\pi N T}{550} \times 746$$

$$\text{but } "N" = \frac{2f}{\#\ poles}$$

$$\therefore \text{"}T\text{"} = \frac{T_{syn} \times 550}{2\pi\, 746} \times \frac{P}{f} \quad \text{lbs. ft. per phase}$$

$$= .117\, T_{syn} \frac{P}{f} \quad \text{lbs. ft. per phase}$$

Mechanical P = T_{syn} x Speed

or equals $\frac{2\pi T N}{550}$ H. P.

where T = lbs.ft. and N = R.R.S.

Therefore efficiency = $\frac{P_{mech} - friction}{e^2(b_1c_1 + b_2c_2)}$

Consider as an example an induction motor, rated at 5 H.P. at 900 R.P.M., on a 110 volt circuit, at 60 ∿. Assume that the machine is an 8 pole machine Δ wound; that

r_s = .25 ohms x_s = 1.00 ohms b = .058

r_p = .20 " x_p = 1.2 " g = .0073

The necessary values can be found by complex quantities for no slip and total slip as tabulated below, and other intermediate values taken from a circle diagram. The results are as follows.

CIRCLE DIAGRAM of Induction Motor

Scale

1 c.m = 2 amperes

1 c.m = 222 watts.

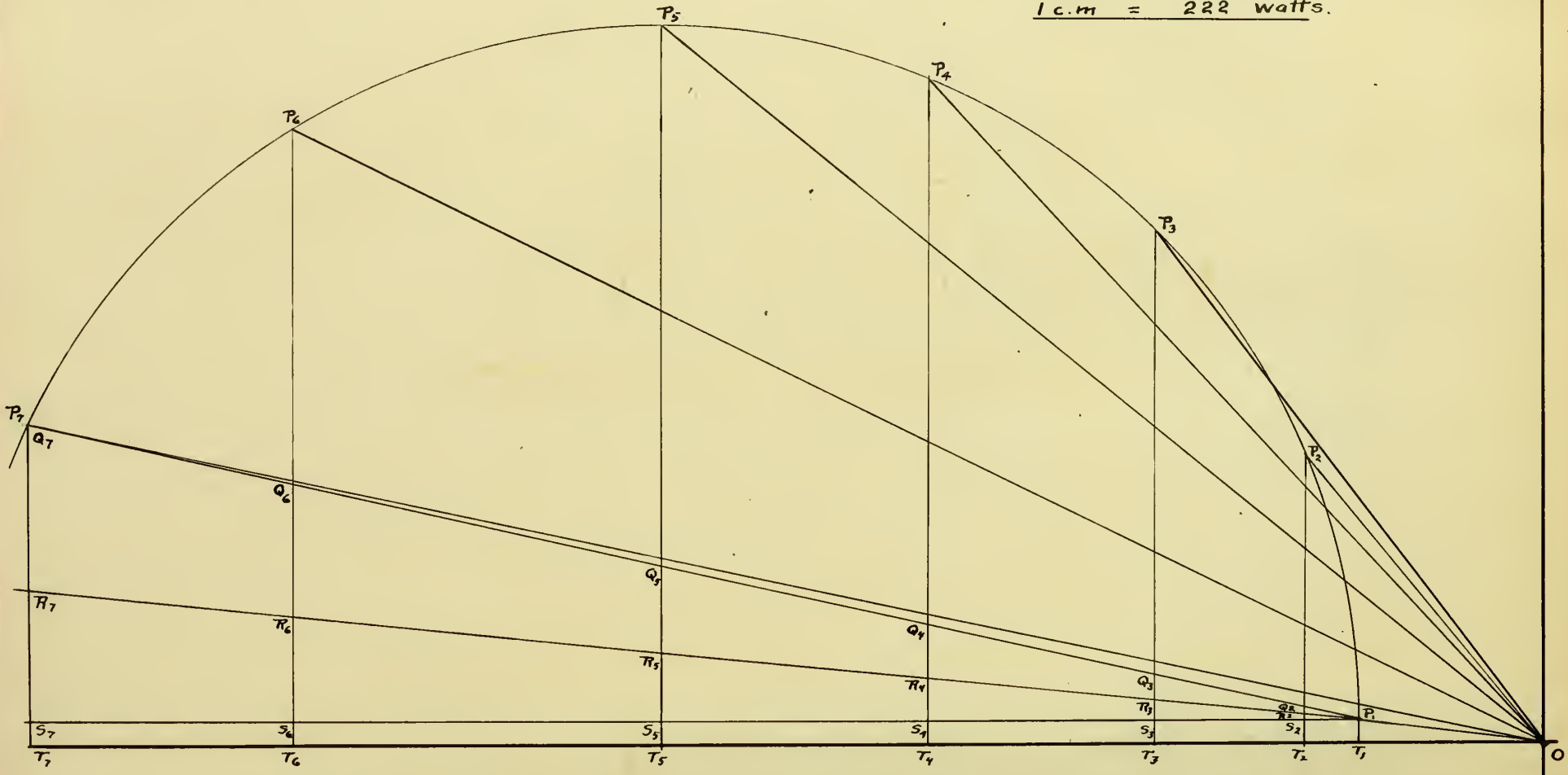


Diagram #18

Slip	0	1	Slip	0	1
$r_s^2 + s^2 x_s^2$	.0625	1.0625	$a_1^2 + a_2^2$	0	.972
sr_s	0	.25	I_s	0	47.6
a_1	0	.235	$b^2 + b^2$	.0586	1.03
$s^2 x_s$	0	1.00	I_p	6.02	50.5
a_2	0	.942	e^2	10550	2545
b_1	.0073	.2423	$b_1 c_1$	.00784	.545
b_2	.058	1.000	$b_2 c_2$	.00016	-.091
$b_1 r_p$	.00146	.048	$e (bc_1 + bc_2)$	84.5	1155
$b_2 x_p$	.0696	1.2	$E_x I_p$	662	5555
c_1	1.0710	2.248	Speed	900	0
$b_2 r_p$	.01160	.20	T_{syn}	0	598
$b_1 x_p$	.00876	.291	T	0	14
c_2	.00284	-.091	Output	0	0
$\sqrt{c_1^2 + c_2^2}$	1.072	2.25	$P_m - f$	0	0
E_x	110	110	Efficiency	0	0
e	102.6	49	Power Factor	.1275	.208

Using the values for no slip and total slip as a basis for a circle diagram, the intermediate values can readily be obtained from the diagram: refer to diagram #18, page 45.

In the circle diagram,

O P equals Current

P T " Power input

P Q " " output

Q R " Secondary I^2R

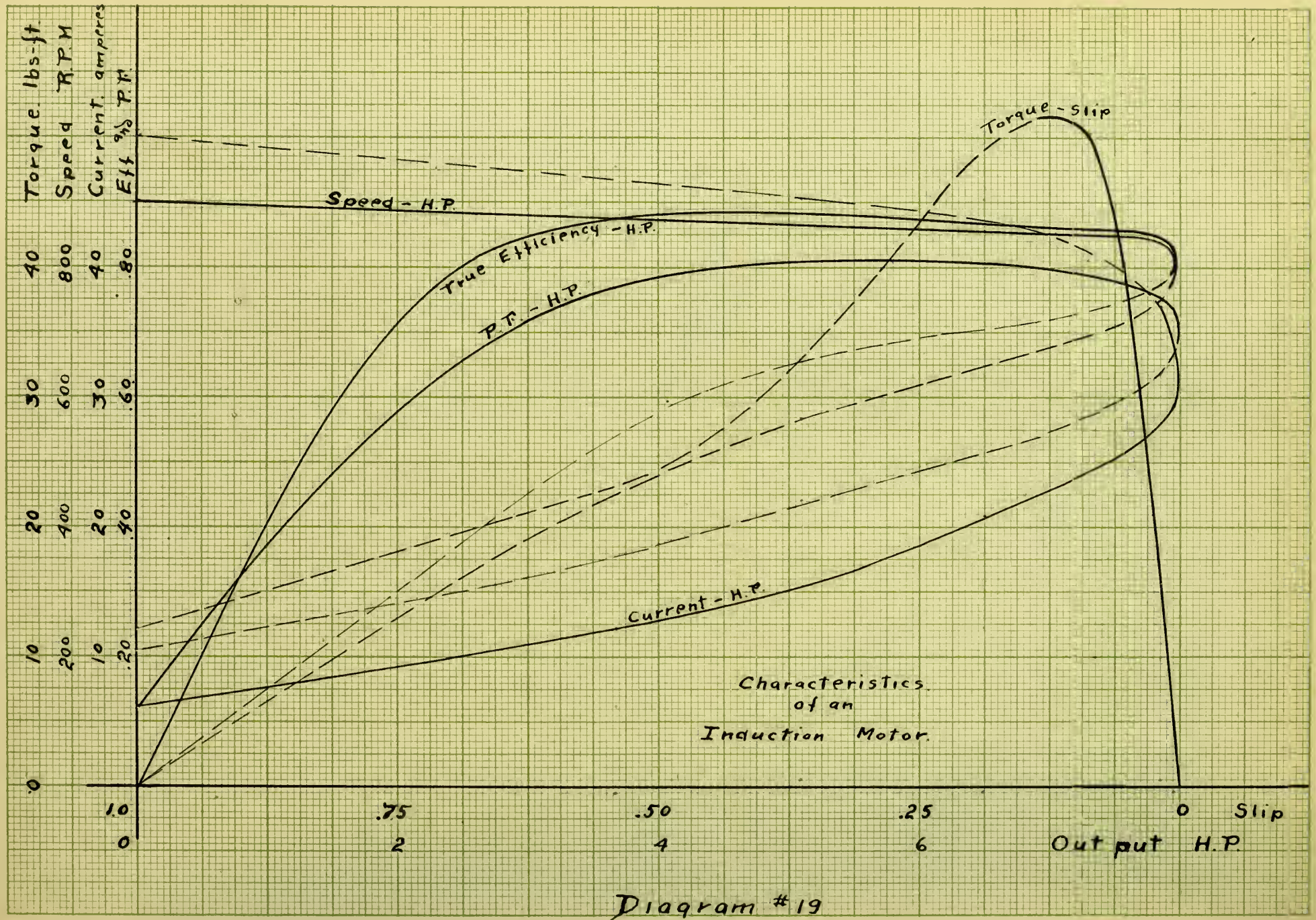

Torque. lbs-ft.
Speed R.P.M
Current. amperes
Eff % P.F.
Torque - Slip
Speed - H.P.
True Efficiency - H.P.
P.F. - H.P.
Current - H.P.
Characteristics of an Induction Motor.
40
800
40
.80
30
600
30
.60
20
400
20
.40
10
200
10
.20
0
1.0
.75
.50
.25
0
Slip
0
2
4
6
Out put H.P.

Diagram #19

R S equals Primary I R

P R " Torque in Syn. Watts

$\frac{Q\ R}{R\ P}$ " Slip

$\frac{R\ Q}{P\ R}$ " Speed

$\frac{P\ Q}{P\ T}$ " Efficiency

Measuring these values to scale from diagram #18, the following results are obtained.

P	1	2	3	4	5	6	7
I_p	6.02	12.2	20.9	29.30	37	45.2	50.5
Output/ phase	0	.910	1.61	1.975	1.95	1.265	0
Input/ phase	.0845	1.042	1.865	2.40	2.6	2.22	1.155
Efficiency	0	.875	.864	.824	.75	.57	0
Slip	0	.0264	.0544	.0856	.1365	.276	1.
Speed	900	876	852	823	777	652	0
Power Factor	.127	.77	.804	.738	.633	.442	.208
Torque Syn	0	.935	1.705	2.16	2.26	1.745	.598
Torque #ft	0	21.9	40	50.6	53	40.8	14
Power H.P.	0	3.66	6.48	7.95	7.85	5.09	0
Energy I_p	.76	9.4	16.8	21.6	23.4	19.95	10.5

Plotting Torque in #ft. against slip; and current, efficiency, power factor, and speed against output, gives the characteristics

of the induction motor considered. Refer to diagram #19, page 47.

The starting torque of an induction motor depends upon the current flowing in the secondary windings, the flux, and the number of rotor conductors. The current in the secondary depends upon the resistance of the conductors, and the induced voltage.

It can be proven that by increasing the resistance of the rotor circuit, the maximum torque will occur at a greater slip and the torque at total slip will increase, so that it is evident that there will be a value of resistance such that the maximum torque will occur at starting. This value of resistance is found to be a value equal to the rest of the impedance of the machine. The values of starting torque can be found for the same machine by increasing the secondary resistance, and the relation between current and starting torque determined. The results are as follows.

r_s	.1	.25	.50	.75	1.
a_1	.099	.235	.40	.48	.5
a_2	.99	.942	.80	.64	.5
b_1	.1063	.2423	.4073	.4873	.5073
b_2	1.048	1.000	.858	.698	.558
c_1	2.279	2.25	2.11	1.94	1.77
c_2	.082	-.091	-.3164	-.4454	-.50
E_x	110	110	110	110	110
e	48.25	49	51.6	55.3	59.8
I_s	48.25	47.6	46.1	44.25	42.25
I_p	50.9	50.5	49.	47	45
e^2	2330	2545	2660	3060	3580
$e\ (b_1 c_1 + b_2 c_2)$	765	1155	1565	1940	2220
$E_x I_p$	5600	5555	5400	5170	4950
T_{syn}	231	598	1063	1470	1790
T	5.4	14	24.9	34.4	42
P. F.	.1365	.208	.29	.376	.448
I_s(energy)	6.6	9.9	13.4	16.6	18.9

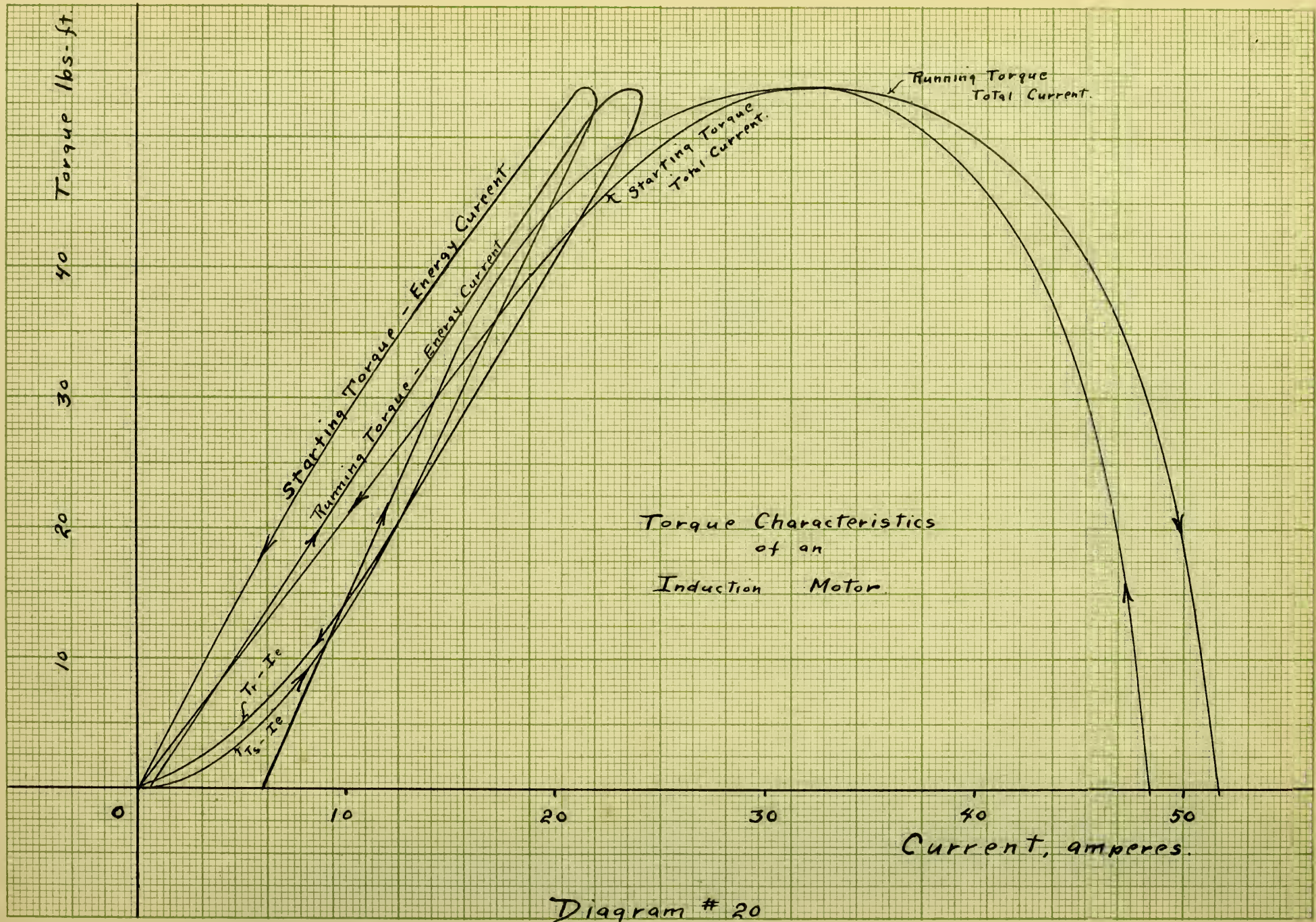

Torque lbs-ft.
10
20
30
40
0
10
20
30
40
50
Current, amperes.
Running Torque
Total Current.
Starting Torque
Total Current.
Starting Torque - Energy Current.
Running Torque - Energy Current
Tr - Ie
Ts - Ie
Torque Characteristics
of an
Induction Motor

Diagram # 20

r_s	1.5	2.	2.5	3.	5.
a_1	.462	.4	.345	.3	.192
a_2	.307	.2	.138	.1	.0384
b_1	.4693	.4073	.3523	.3073	.1993
b_2	.365	.258	.196	.158	.0964
c_1	1.542	1.39	1.3	1.25	1.155
c_2	-.491	-.4374	-.385	-.3374	-.221
E_x	110	110	110	110	110
e	68	75.7	81	85.4	93.8
I_s	41.2	33.8	30.1	27	18.4
I_p	40.5	36.5	32.6	29.4	20.7
e^2	4640	5740	6580	7300	8820
$e\ (bc_1 + bc_2)$	2580	2600	2530	2410	1840
$E_x I_p$	4550	4020	3580	3240	2280
T_{syn}	2140	2300	2270	2190	1690
T	50	53.8	53.2	51.3	39.6
P.F.	.567	.648	.708	.745	.808
I_s(energy)	23.4	21.9	21.3	20.1	14.85

The relation between starting torque and running torque with current, as plotted from the above data and data on page 48, is shown on diagram #20, page 51.

From the data at hand, that is, with starting torque, normal speed, and input, it is possible to calculate the torque efficiency and compare it with true efficiency. The calculations are tabulated as follows.

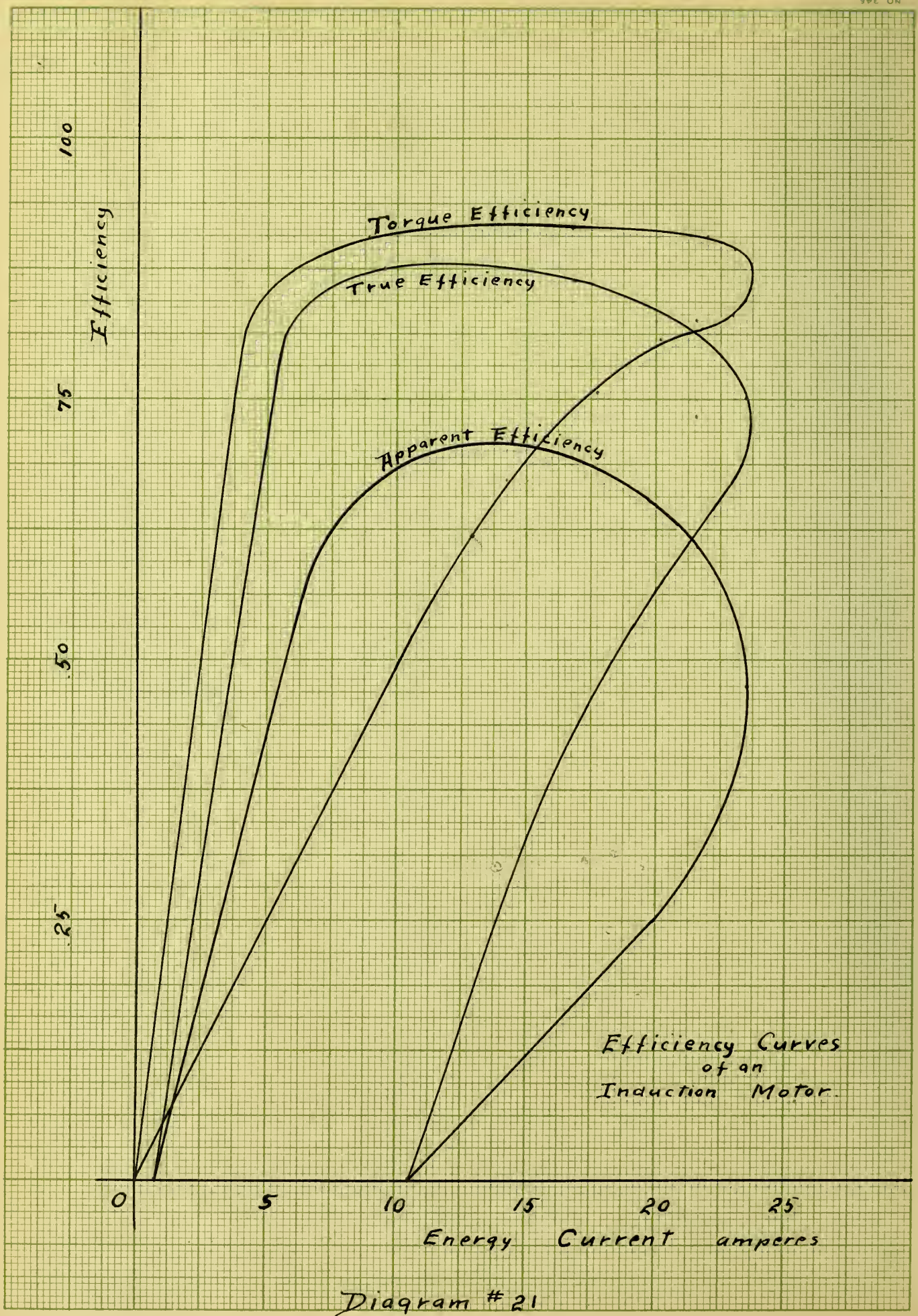

Efficiency
100
75
50
25
Torque Efficiency
True Efficiency
Apparent Efficiency
0
5
10
15
20
25
Energy Current amperes
Efficiency Curves
of an
Induction Motor.

Diagram # 21

T_s	S_n	$T_s \times S_n \times 2\pi \frac{746}{550}$	$E_1 I \cos \Theta$	Torque Eff	I_{energy}
22.5	15	2.87	3.174	.904	8.9
39.6	15	5.07	5.55	.913	14.85
51.3	15	6.58	7.25	.908	20.1
53.2	15	6.83	7.62	.897	21.3
53.8	15	6.9	7.8	.884	21.9
50	15	6.4	7.73	.83	23.4
42	15	5.38	6.66	.806	18.9
34.4	15	4.41	5.83	.756	16.6
24.9	15	3.19	4.70	.658	13.4
14	15	1.795	3.47	.518	9.9
5.4	15	.692	2.13	.325	6.6

Plotting torque efficiency, apparent efficiency, and true efficiency against current as shown on diagram #21, page 53, the relation between them is readily seen.

The torque characteristics and its speed characteristic show that the induction motor is applicable to service of a similar nature to that of the shunt d. c. motor. The induction motor has almost constant speed through the working range of the machine. The speed can be varied to any desired value but at the expense of the efficiency of operation. The advantageousness of the induction motor is, that it is used on alternating current circuits, and the transmission of alternating current is always more economical than direct current.

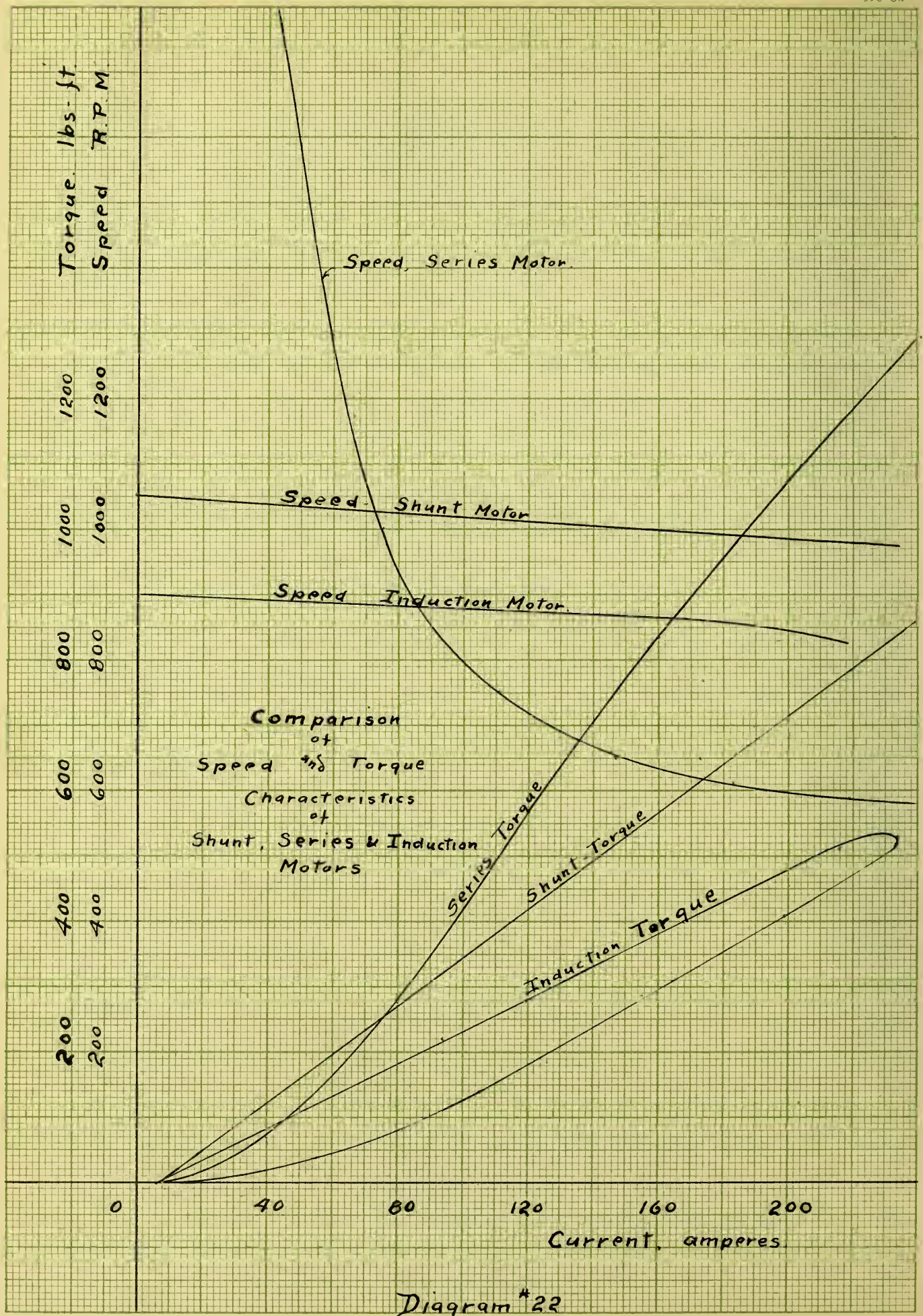

Torque. lbs-ft.
Speed R.P.M.
1200
1200
1000
1000
800
800
600
600
400
400
200
200
0
40
80
120
160
200
Current, amperes.
Speed, Series Motor.
Speed Shunt Motor
Speed Induction Motor.
Comparison of Speed and Torque Characteristics of Shunt, Series & Induction Motors
Series Torque
Shunt Torque
Induction Torque

Diagram #22

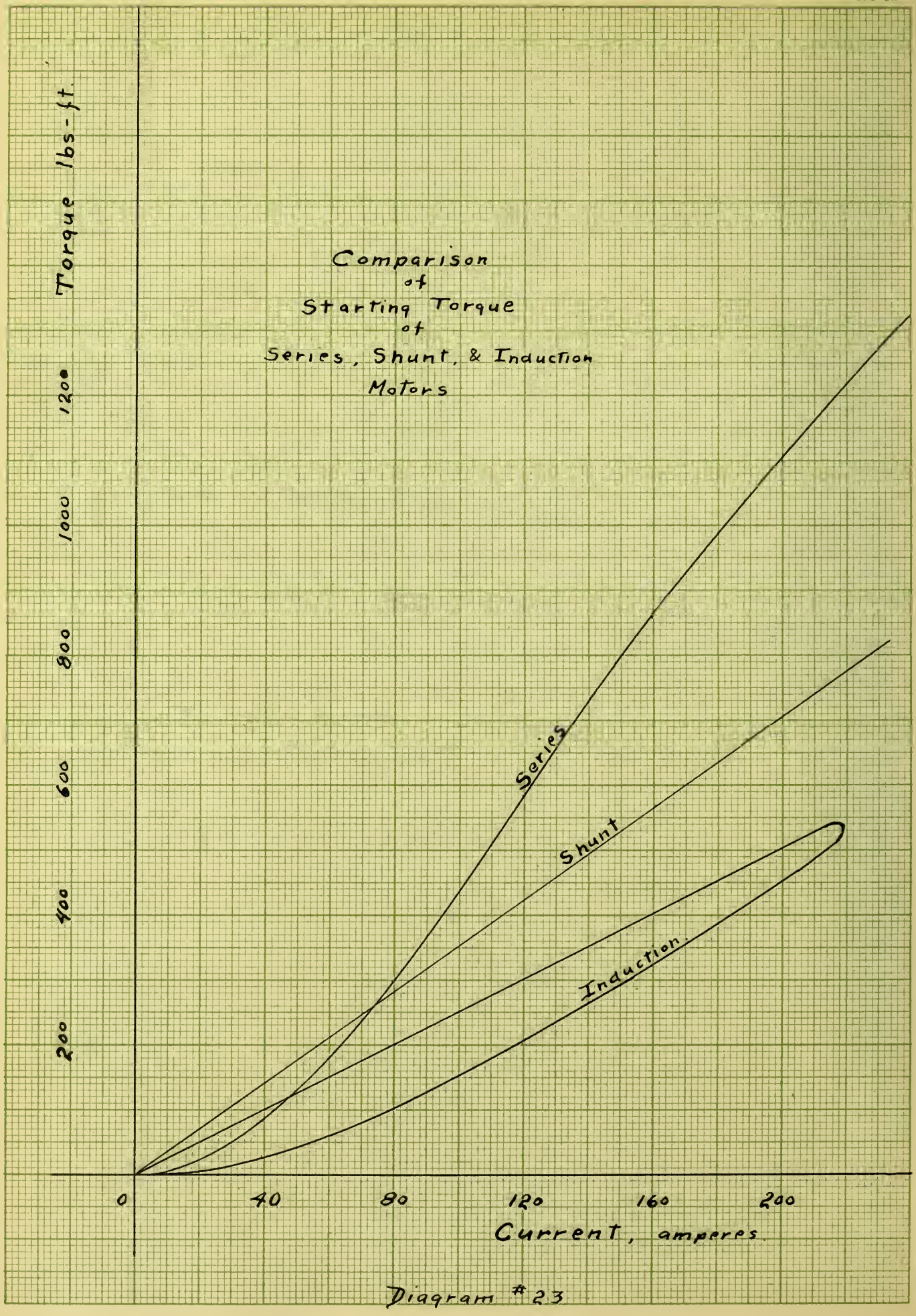

Comparison
of
Starting Torque
of
Series, Shunt, & Induction
Motors
Torque lbs-ft.
1200
1000
800
600
400
200
Series
Shunt
Induction
0
40
80
120
160
200
Current, amperes.

Diagram #23

SUMMARY

By plotting the speed and torque curves to the same scale as shown by diagram #22, page 55, it is possible to compare the three types of motors considered, in regard to those characteristics. It is seen that the torque of the series motor increases much more rapidly for a given amount of current, except for very small values, than does the shunt or induction motor; while a study of its speed curve shows a rapid falling off as torque increases. The speed gradually comes to an almost constant value after the magnetic field becomes saturated. After this point is reached, the motor would tend to run similarly to a shunt motor, but it occurs beyond the working range of the machine. The torque characteristics of the shunt and induction motors are in general similar, and their speed curves are practically the same, so that in general they could be used on the same class of service. But there are a good many other considerations that determine the choice of one or the other that do not come within the scope of this thesis, such as various methods of speed variations, etc.

The starting torque curves as shown on diagram #23, page 56, show the same differences as did the running torques of the same machines.

From the efficiency curves as shown under each chapter, it was seen that the torque efficiency is in all cases higher than the true efficiency. True efficiency is based upon running torque and speed for its output, while torque efficiency is

based upon starting torque and speed. Since starting torque is in general higher for any value of current than running torque, the torque efficiency therefore must be higher.

The practical application of these types of motors depends upon their torque and speed relations. The shunt motor, due to its constant speed, is used for such service as driving shafting or individual machines in manufacturing establishments where constant speed is required. The induction motor is also coming into this class of service, but its chief advantage is due to the fact that the induction motor operates on alternating current, which is more economically transmitted any distance to a factory than is the direct current. The series motor is used in all classes of service where torque rather than speed is of importance. This class of service is found in cranes and electric railway cars. It is essential that the load be rigidly attached to the series motor, for on light loads the series motor will race.

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